However, the hierarchy of keys between an authority and the accredited entities is not obligatory. One can use GQ1 with a module belonging to the proving entity, which makes it possible to use the Chinese remainder technique to reduce the work loads of the proving entity, which does not change the work load of the checking entity fundamentally, apart from the fact that a modulus at the level of the proving entity can be shorter than a modulus at the level of the authority, for example 512 bits in comparison with 768 bits.

When the entity knows the prime factors of its own modulus, why call on an RSA digital signature mechanism??

Another version of GQ methods, here called elementary GQ2, treats directly the problem of factorization of a modulus n. In this context, "directly" means "without calling for the RSA signature". The aim of GQ2 is to reduce work loads, not only of the proving entity but also of the checking entity. The proving entity demonstrates knowledge of a breakdown of its own modulus and this proof does not reveal the breakdown which therefore remains secret, so as to be able to be used as often as needed. The security of the GQ2 protocol is equivalent to factorization of the module.

Each proving entity has its own modulus n. Each GQ2 mechanism implements a parameter k, a small number greater than 1 fixing a public exponent $v=2^k$, and one or several pairs of numbers $\{G_1,Q_1\}$ to $\{G_m,Q_m\}$. Each public number G_i is the square of a small number g_i greater than 1 and called a "base number". All the proving entities can use the same public number or numbers G_1 to G_m. The factorization of the modulus n and the private number or numbers Q_1 to Q_m are then at the same level in the key hierarchy. Each set of elementary GQ2 keys is defined by two necessary and sufficient conditions:

- for each base number, neither of the two equations $x^2 \equiv \pm g_i \pmod{n}$ does not have a solution in x in the ring of integers modulo n, that is to say that the numbers $\pm g_i$ are two non-quadratic residues modulo n.

- for each base number, the equation $x^v \equiv g_i^2 \pmod{n}$ where $v=2^k$ has solutions in x in the ring of integers modulo n. The private number Qi or its inverse modulo n is any one of these solutions.

Taking into account the second condition, in order that the numbers $\pm g_i$ are two non-quadratic residues modulo n, the modulus n must comprise at least two prime factors congruent to 3 (mod 4) relative to which the Legendre symbol of g_i differs. Consequently, any modulus composed of prime factors of which none or a single one is congruent to 3 (mod 4) does not allow a set of elementary GQ2 keys to be established, which privileges the prime factors congruent to 3 (mod 4). Thus, by taking high prime numbers at random, it appears that only half are congruent to 3 (mod 4) and half to 1 (mod 4). As a result, many RSA models in use cannot establish a set of elementary GQ2 keys.

Here we introduce the sets of generalised GQ2 keys to overcome this limitation in order to be able to use GQ2 technology with any modulus whatsoever, and in particular with any RSA modulus whatsoever: they depend on two necessary and sufficient principles.

The first principle reproduces the second elementary GQ2 condition:

- For each base number g_1 to g_m, the equation $x^v \equiv g_i^2 \pmod{n}$ where $v=2^k$ has solutions in x in the ring of integers modulo n.

Because the private number Q or its inverse modulo n is a solution to the equation, successive squares k-1 modulo n transform it into a number q_i which is a square root of Gi in the ring of integers modulo n. Depending on whether the number q_i is equal to one of the two numbers g_i or n-g_i, or different from the two numbers g_i and n-g_i, it is said that it is trivial or not. When a number q_i is non trivial, n which divides q_i^2-g_i^2 does not divide either q_i-g_i or q_i+g_i. Any non trivial number q thus reveals a breakdown of the modulus n.

$$n=\mathrm{pgcd}(n,q_i-g_i)\mathrm{x}\,\mathrm{pgcd}(n,q_i+g_i)$$

The second principle widens the first elementary GQ2 condition:

- among the numbers q_1 to q_m, at least one number q_i is non trivial.

It is to be noted that if a number q+ exists while the numbers $\pm g_i$ are two non-quadratic residues in the ring of integers modulo n, the number q_i is manifestly non trivial. Thus, the set of elementary GQ2 keys are certainly part of the set of generalised GQ2 keys which make it possible to use any modulus whatsoever, that is to say any composition of high prime numbers indifferently congruent to 3 or to 1 (mod 4) at least two of which are distinct. On the other hand, many sets of generalised GQ2 keys are not sets of elementary GQ2 keys. Each set of generalised GQ2 keys is in accordance with one of the following cases:

- when the 2xm numbers $\pm g_1$ to $\pm g_m$ are all non-quadratic residues, this is a set of elementary GQ2 keys.

- when among the 2xm numbers $\pm g1$ to $\pm gm$, there is at least one quadratic residue, this is not a set of elementary GQ2 keys, it is what is known here as a set of complementary GQ2 keys.

The present invention relates to the sets of complementary GQ2 keys, by definition these sets of generalised GQ2 keys not being elementary. Besides the two preceding principles, such a set must satisfy a third principle.

-among the 2xm numbers $\pm g_1$ to $\pm g_m$, there is at least one quadratic residue.

In order to assess the problem and understand the solution we provide, that is to say the invention, let us first of all analyse the breakdown of the modulus n revealed by a non trivial number q, and then recall the Chinese remainder technique, and then the notion of rank in a Galois field CG(p); and next, study the functions "raise to the square" in CG(p) and "take a square root" of a quadratic residue in CG(p); and

finally, analyse the applicability of the three principles stated above.

Analysis of the breakdowns of the module :

Just as the modulus n is broken down into f prime factors p_1 to p_f, the ring of integers modulo n is broken down into f Galois fields $CG(p_1)$ to $CG(p_f)$. In each field, there are two unit square roots, that is ± 1. In the ring, there are thus 2^f unit square roots. Each private number Q_1 to Q_m defines a number $\Delta_i = q_i/g_i \pmod{n}$ which is one of these 2^f unit square roots in the ring; in other words, n divides $\Delta_i^2 - 1$.

• when q_i is trivial, that is to say $\Delta_i = \pm 1$, n divides $\Delta_i - 1$, or $\Delta_i + 1$ and therefore Δ_i does not reveal the breakdown of the modulus n.

• when q_i is non trivial, that is to say $\Delta_i \Delta \pm 1$, n does not divide either $\Delta_i - 1$ or $\Delta_i + 1$ and thus Δ reveals a breakdown, $n = \text{pgcd}(n, \Delta_i - 1) \times \text{pgcd}(n, \Delta_i + 1)$, resulting from the value of Δ_i in each field : the prime factor or factors dividing $\Delta_i - 1$ on the one hand, this or these dividing $\Delta_i + 1$ on the other.

Examination of the rules of multiplicative composition of the numbers q. Two numbers $\{q_1, q_2\}$ give a composite number $q_1 \times q_2 \pmod{n}$.

- when q_1 is non trivial and q_2 is trivial, the composite number $q_1 \times q_2 \pmod{n}$ is non trivial; it reveals the same breakdown as q_1.

- when q_1 and q_2 are non trivial and $\Delta_1 = \pm \Delta_2$ The composite number $q_1 \times q_2 \pmod{n}$ is trivial; it does not reveal any breakdown.

- when q_1 and q_2 are non trivial and $\Delta 1 \neq \pm \Delta 2$, the composite number $q_1 \times q_2 \pmod{n}$ is non trivial; it reveals a third breakdown.

Three numbers $\{q_1, q_2, q_3\}$ give four composite numbers $\{q1xq2, q1xq3, q2xq3, q1xq2xq3 \pmod{n}\}$, that is a total of seven numbers; m numbers thus provide $2^m - m - 1$ composite numbers, that is a total of $2^m - 1$ numbers.

We shall now consider a set of generalised GQ2 keys comprising i base numbers g_1 to g_i and i private numbers Q_1 to Q_i giving i numbers q_1 to q_i and therefore i numbers Δ_1 to Δ_i which are the unit roots. Let us try to take into account another base number g_{i+1} by a private number Q_{i+1} giving a number q_{i+1} and thus a root Δ_{i+1}.

• The total of the 2^{i+1}-1 numbers comprises as many non trivial numbers in each of the following cases.

- the root Δ_{i+1} is trivial and at least one root Δ_1 to Δ_i is non trivial.

- The root Δ_{i+1} is non trivial and figures among the 2xi roots $\pm\Delta_1$ and $\pm\Delta_i$.

• In the case where the root Δ_{i+1} is non trivial and does not figure among the 2xi roots $\pm\Delta_1$ to $\pm\Delta_i$, each composite number where q_{i+1} figures is non trivial.

Consequently, when among the m numbers q_1 to q_m, at least one is non trivial, more than half the total of the 2^m-1 numbers are non trivial.

By definition, it is said that $l<f$ non trivial numbers $\{q_1, q_2, \ldots q_l\}$ are independent relative to the modulus n when each of the 2^l-l-1 corresponding composite numbers is non trivial, that is to say that, in total, the 2^l-1 numbers are all non trivial. Each of these 2^l-1 numbers thus reveals a different breakdown of the modulus n.

- When the f prime factors are distinct, there are 2^{f-1}-1 breakdowns of the modulus n. Thus, if f-1 numbers q are independent, there is a bi-univocal correspondence between the $2^{f-1}-1$ breakdowns and a total of $2^{f-1}-1$ numbers comprising the f-1 independent numbers and the $2^{f-1}-f$ corresponding composite numbers.

• **Chinese remainders**

Either two numbers a and b, prime numbers between themselves such as $0<a<b$, and two numbers X_a from 0 to a-1 and X_b from 0 to b-1; it concerns the determination of the unique number X from 0 to axb-1 such that $X_a \equiv X (\mathrm{mod}\ a)$ and

$X_b \equiv X(\text{mod } b)$. The number $x \equiv \{b(\text{mod } a)\}^{-1}(\text{mod } a)$ is the Chinese remainders parameter. The elementary Chinese remainders operation is given below:

$x \equiv X_b$ (mod a)

$y = X_a - x$; if y is negative, replace y by $y+a$

$z \equiv$ áxy (mod a)

$X = z \times b + X_b$

To resume, one writes : X = Chinese Remainders (X_a, X_b).

When f prime factors are arranged in increasing order, from the smallest p_1 to the biggest p_f, the Chinese remainder parameters can be the following (there is one less than the prime factors, that is to say f-1).

- The first parameter is $x \equiv (p_2(\text{mod } p_1))^{-1}(\text{mod} p_1)$.
- The second parameter is $\beta \equiv (p_1 x p_2(\text{mod} p_3))^{-1}(\text{mod} p_3)$.
- The i-th parameter is $\lambda \equiv (p_1 \text{x} \ldots p_{i-1}(\text{mod } p_i))^{-1}(\text{mod } p_i)$.
- And so on.

In f-1 elementary operations, one establishes a number X from 0 to n-1 starting from any set of f components from X_1 to X_f with X_j from 0 to p_j-1 :

- a first result (mod $p_1 \text{x} p_2$) with the first parameter,
- then a second result (mod $p_1 x p_2 x p_3$) with the second parameter,
- until the final result (mod $n = p_1 x p_2 x \ldots p_f$) with the last parameter.

To resume, given the prime factors p_1 to p_f, each element of the ring of integers modulo n has two equivalent representations:

- f numbers X_1 to X_f, a prime factor component: $X_j \equiv X(\text{mod} p_j)$,
- a number X from 0 to n-1, X=Chinese remainders $(X_1, X_2, \ldots X_f)$.

Rank of numbers in CG(p) - Let p be an odd prime number and a a number smaller than p i.e. $0 < a < p$. By definition, the rank of a with respect to p is the period of the sequence $\{X\}$ defined by $\{x_i = a$; then, for $i \geq 1$, $x_{i+1} \equiv a \times x_i$ (mod

$p)\}$. By applying the Fermat theorem we obtain: $x_{i+p} \equiv a^p \times x_i \equiv a \times x_i \equiv x_{i+1} \pmod{p}$. Therefore, the rank of a number a with respect to a prime number p is p-1 or a divider of p-1.

For example, when $(p-1)/2$ is an odd prime number p', the Galois field CG(p) includes a number of rank 1: this is 1, a number of rank 2: this is -1, p'-1 numbers of rank p' and p'-1 numbers of rank $2 \times p' = p-1$. In CG(p), any number of rank p-1 is a "generator". The name is due to the fact that the successive powers of a generator in CG(p), i.e. the terms of the sequence $\{X\}$ for indices from 1 to p'-1, form a permutation of all the non-zero elements of CG(p).

Let y be a generator of CG(p). Let us evaluate the rank of the number y' (mod y) according to i and p-1. When i is prime with p-1, this is p-1. When i divides p-1 this is $(p-1)/i$. In all cases, this is $(p-1)/\mathrm{lgcd}(p-1,i)$ (lgcd = largest common divisor).

By definition, Euler's function $\varphi(n)$ is the number of numbers smaller than n and prime with n. In CG(p), there are $\varphi(p-1)$generators.

As an illustration, understanding the bases of the RSA is facilitated with the rank. Module n is the product of f prime factors p-1 to p-f, with $f \geq 2$. For each prime factor p_i from p_1 *to* p_f the public exponent e should be prime with p_j-1. Now, key $<e,p_j>$ observes the rank of the elements of CG(p_j): it permutates the elements of CG(p_j); there exists a number d_j, generally as small as possible, such that p_{-1} divides $e \times d_{j-1}$. The key $<d_j, p_j>$ inverts the permutation of the elements of CG(p_j). These f permutations, one in each field CG(p_1) to CG(p_f), are expressed in the ring of integers modulo n by the RSA permutation summarized by the public key $<e, n>$. There exists a number d, generally as small as possible, such that the smallest common multiple (scm) of $(p_{1-}, p_{2-1}, \ldots p_{f-1})$ divides $d \times e$-1. For each prime factor p_j from p_1 to p_f, we have $d_j \equiv d$ (mod p_{j-1}). The RSA permutation summarized by the public key $<e, n>$ is inverted by the private key $<d, n>$.

Squares in CG(p) - Let us define a number t such that p-1 is dividable by 2^t, but not by 2^{t-1}. Each large prime number appears in one and only one category: $t = 1$, $t = 2$, $t = 3$, $t = 4$, and so forth. If a sufficiently large number of successive prime numbers are considered, about one out of two appears in the first category where p is congruent to 3 (mod 4), one out of four in the second one, where p is congruent to 5 (mod 8), one out of height in the third one, where p is congruent to 9 (mod 16), one out of sixteen in the fourth one, where p is congruent to 17 (mod 32), and so forth; on the average, one out of 2^t appears in the nth category where p is congruent to 2^{t+1} (mod 2^{t+1}).

Because numbers x and p-x have the same square in CG(p), key <2,p> does not permutate CG(p). The function "take the square" in CG(p) may be represented by an orientated graph wherein each non-zero element of the field finds its place. Let us analyze the structure of the graph into branches and cycles according to the parity of the rank of each element.

- <u>The zero element is set.</u> This is 0. Rank is not defined for the zero element to which no other element is related; the zero element is isolated.

- <u>The unit element is set.</u> This is 1, the only element of rank 1. All the roots of unity in CG(p) are located in the branch related to 1. Let y be a non-quadratic residue of CG(p), any residue; key <p-1)/2^t, p> transforms y into a primitive 2^{t-1}th root of -1 referenced as b; indeed, we have $y^{(p-1)/2} \equiv -1 \pmod{p}$. Therefore, in CG($p$), the powers of b for exponents from 1 to 2^{t-1}, are the 2^{t-1} roots of unity other than 1: they make up the branch related to 1.

- <u>The square of any element of even rank is another element the rank of which is divided by two.</u> Therefore, each element of even rank is placed in a branch; each branch includes a rank number dividable by two, but not by four, next, if $t \geq 2$, two rank numbers dividable by four but not by

eight, next, if $t \geq 3$, four rank numbers dividable by eight but not by sixteen, next, if $t \geq 4$, eight rank numbers dividable by sixteen but not by 32, and so forth. All the branches are similar to the branch related to 1; the 2^{t-1} leaves of each branch are non-quadratic residues; each branch includes 2^{t-1} elements and is related to an element of odd rank; there are $(p-1)/2^t$ branches which are all of the same length t.

- <u>The square of any element of odd rank other than the unit element is another element having the same rank.</u> Key <2,p> permutates the set of $(p-1)/2^t$ elements of odd rank. The permutation is factorized into permutation cycles. The number of cycles depends on the factorization of $(p-1)/2^t$. For each divider p' of $(p-1)/2^t$, there is a cycle including the $\varphi(p')$ elements of rank p'. Let us recall that by definition, Euler's function $\varphi(p')$ is the number of numbers smaller than p' and prime with p'. For example, when p' equals $(p-1)/2^t$, is prime, the p'-1 numbers of rank p' form a large permutation cycle.

Figures 1A-1D each illustrate a graph fragment for p congruent to 3 (mod 4), 5 (mod 8), 9 (mod 16) and 17 (mod 32), respectively.

- The leaves on the branches are illustrated by white circles; these are non-quadratic residues.
- The nodes in the branches are illustrated by grey circles; these are quadratic elements of even rank.
- The nodes in the cycles are illustrated as black circles; these are quadratic elements of even rank.

Square roots in CG(p) - Knowing that a is a quadratic residue of CG(p), let us see how to calculate a solution to the equation $x^2 \equiv a \pmod{p}$ i.e. "take a square root" in CG(p). Of course, there are many ways for obtaining the same results; pages 31-36 of the book of Henri Cohen, *a Course in Computation Algebraic Number Theory*, published in 1993 by Springer in Berlin as volume 138 of the *Graduate Texts in Mathematics* series (GTM 138), may be consulted.

The number $s = (p-1+2^t)/2^{t+1}$ provides a key $<s, p>$) which is:

$<(p+1)/4,\ p>$ when p is congruent to 3 (mod4),
$<(p+3)/8,\ p>$ when p is congruent to 5 (mod8),
$<(p+7)/16,\ p>$ when p is congruent to 9 (mod16),
$<(p+15)/32,\ p>$ when p is congruent to 17 (mod32),
and so forth.

- Key $<s, p>$ transforms any element of a cycle into the previous element in the cycle. When a is of even rank, it is the solution of odd rank, we name it w. Indeed, in CG(p), w^2/a is equal to a to the power of $(2\times(p-1+2^t)/<2^{t+1})-1 = (p-1)/2^t$. The other solution is of even rank; this is $p-w$.

- Generally, the key $<s, p>$ transforms any quadratic residue a in a first approximation, into a solution which we name r. As a is a quadratic residue, the key $<2^{t-1}, p>$ certainly transforms r^2/a into 1. To approach a square root of a, let us take the power 2^{t-2} of r^2/a (mod p) in order to obtain +1 or -1. The new approximation remains r if the result is +1 or else it becomes $b \times r$ (mod p) if the result is -1, knowing that p refers to any primitive 2^tth root of 1 in the field CG(p). Therefore, the key $<2^{t-2}, p>$ transforms the new approximation into 1. It may still be approached by using the key $<2^{t-3}, p>$ and by multiplying with $b2$ (mod p) if necessary, and so forth.

The following algorithm solves the equation. It uses numbers a, b, p, r and t, as defined above and two variables: c represents the successive corrections and w the successive approximations. At the beginning of the algorithm, $c = b$ and $w = r$. At the end of the calculation, the two solutions are w and $p-w$.

For i from t-2 to 1, repeat the following sequence:

- Apply key $<2^i, p>$ to number w^3/a (mod p) in order to obtain +1 or -1.
- When -1 is obtained, replace w by $w\times c$ (mod p).
- Replace c by $c2$ (mod p).

Applicability of the principles - By definition we state that a parameter k, a base number g and a prime factor p are *compatible* when the equation $x^{v} \equiv g^2 \pmod{p}$ where exponent v is 2^k, has solutions in x in the field CG(p). Numbers k and g are small and larger than 1. Number p is a large prime number.

- When $t = 1$, i.e. $p \equiv 3 \pmod 4$, the equation has two solutions.

- When $t = 2$, i.e. $p \equiv 5 \pmod 8$, according to the Legendre symbol of g with respect to p, the equation has four solutions if $(g \mid p) = +1$; it does not have any solution if $(g \mid p) = -1$.

- When $t > 2$, i.e. $p \equiv 1 \pmod 8$, let u be the number such that 2^u divides the rank of the public number $G = g^2$ with respect to p, but such that 2^{u+1} does not divide it; therefore, u is equal to one of the numbers from 0 to t-1. The equation has no solution if $u > 0$ and $k + u > t$; it has 2^k solutions if $k + u \leq t$; it has 2^t solutions if $u = 0$ and $k >$ t.

Therefore, there are two types of compatibility according to whether G is in a cycle, or else in a suitable position in a branch.

- When G is in a cycle, i.e. $u = 0$, regardless of the value of k, there is a solution of odd rank in the cycle and solutions of even rank disseminated in $\alpha = \min(k, t)$ consecutive branches related to the cycle, i.e. 2^α solutions in all. Figure 2A illustrates this case with $k \geq t = 3$, i.e. a prime factor congruent to 9 (mod 16), which imposes that $u = 0$.

- When G is in a suitable position in a branch, i.e. $u > 0$ and $u+k \leq t$, there are 2^k solutions, all of even rank and in the branch. Figure 2B illustrates this case.

Given a parameter k, there are therefore two types of prime factors according to whether the value of t is less than k or else larger than or equal to k.

- For any prime factor p_j, such as $t < k$, each G_i should be in a cycle, and there is no solution in the branch

related to G_i. Let us define a number $\Delta_{i,j}$ which is +1 or -1 according to whether g_i or $-g_i$ is in the cycle. There is no choice for any of the m numbers $\Delta_{1,j}$ to $\Delta_{m,j}$. Figure 3A illustrates a case with $t < k$: G_i is in a cycle with a prime factor p_j congruent to 9 (mod 16), i.e. $u = 0$, $t = 3$ with $k > 3$.

- For any prime factor p_j such that $t \geq k$, each G_i should be such that $u + k \leq t$, i.e., either in a cycle with $u = 0$ or else, in a suitable position in a branch with $1 \leq u \leq t-k$. Let us define a number $\Delta_{i,j}$ which is +1 or -1 according to whether Q_{ij} is in the portion of the graph related to g_i or to $-g_i$. There is a choice for each of the m numbers $\Delta_{1,j}$ to $\Delta_{m,j}$; each number $\Delta_{i,j}$ may individually be switched from one value to the other. Figure 3B illustrates a case when $t \geq k$: G_i is in a branch with a prime factor p_j congruent to 17 (mod 32), i.e., $u = 1$, $t = 4$ with $k = 3$.

Each set of f components $\{\Delta_{i,1} \ldots \Delta_{i,f}\}$ is a square root of unity in CG(p_j). This root is trivial or not according to whether the f components are equal or not; we then state that the set of f components is constant or variable, which expresses the fact that the number q_i is either trivial or not. Therefore, when a number q_i is non-trivial, the set of f components $\{\Delta_{i,1} \ldots \Delta_{i,f}\}$ summarizes a factorization of the module. It is therefore possible to test the principles before calculating the private components $Q_{i,j}$.

- When a public number G_i is in a cycle for a prime factor p_j, the number $\Delta_{i,j}$ is +1 or -1, according to whether g_i or $-g_i$ is in the cycle. When $p_j \equiv 3 \pmod 4$, this is Legendre's symbol: $\Delta_{i,j} = (g_i \mid p_j)$.

- When a public number G_i is in a suitable position in a branch for a prime factor p_j, the value to be given to $\Delta_{i,j}$ may be determined by computing the private component $Q_{i,}$.

Production of sets of keys - Given a parameter k, there are two strategies.

- Either the generator requires f prime factors in order to determine m base numbers. The first prime numbers:

2, 3, 5, 7, ... are examined for evaluating their compatibility with each of the f large prime factors p_1 to p_f. Although $g = 2$ is not compatible with $p \equiv 5 \pmod 8$, 2 may enter into the composition of a base number. Indeed, when two numbers are in a similar position in a branch, their product is closer to the cycle, exactly as a square brings the cycle closer. A base number may thereby be obtained by composing the numbers which are not appropriate individually.

- Or the generator requires m base numbers and characteristics of the module such that a bit size (for example, 512, 768, 1024, 1536, 2048) and a number of bits successive to 1 with strong weights (for example, 1, 8, 16, 24, 32) in order to determine $f \geq 2$ prime factors. Noted as G_1, G_2,... G_m, the base numbers generally appear among the first prime numbers: 2, 3, 5, 7, 11 ... or else these are combinations of the first prime numbers. Unless indicated otherwise, these are the first m prime numbers; $G_1 = 2$, $G_2 = 3$, $G_3 = 5$, $G_4 = 7$,... Let us note that $p \equiv 5 \pmod 8$, is not compatible with $g = 2$. Module n will be the product of f prime factors with close sizes, i.e. the size assigned to the module divided by f.

First principle - The parameter k, each prime factor from p_1 to p_f and each base number g from g_1 to g_m should be compatible. Let us define a number h such as 2^h divides the rank of g with respect to p, whereas 2^{h+1} does not divide it. To compute the number h, the following procedure uses Legendre's symbol $(g \mid p)$ and a number b, a primitive 2^tth root of unity in CG(p).

- If $(g \mid p) = +1$ with $t = 1$, return "$h = 0$".
- If $(g \mid p) = +1$ with $t > 1$, apply the key $<(p-1+2^t)/2^{t+1}, p>$ to G in order to obtain a result called w.
- If $w = +g$, return "$h = 0$".
- If $w = p-g$, return "$h = 1$".
- Else, set c to b and for i from t-1 to 2,
- Apply key $<2^i, p>$ to $w/g \pmod p$ in order to obtain $\pm$ 1,

- If -1, set h to i and replace w with $w \times c$ (mod p),
- Replace c with $c2$ (mod p).
- Return "value of h from 2 to t-1".
- If $(g \mid p) = -1$, return "$h = t$".

Let us recall that k, g and p are incompatible when $u > 0$ with $k+u > t$; they are compatible when $h = 0$ or 1, regardless of the value of k, and equally when $k > 1$ with $k+h \leq t+1$.

Second principle - The three following procedures correspond to different implementations of the second principle. In certain implementations, the second principle may be reinforced to the point of requiring that each number q_1 to q_m be not trivial. The role of the base numbers is then balanced; balancing or not the second principle has an effect of certain aspects of the demonstration of the security of the scheme. Finally, when there are $f > 2$ distinct prime factors, among the m numbers $\{q_1 \ldots q_m\}$, it may be required that there be at least one subset of f-1 independent numbers.

The three procedures use $m \times f$ numbers $\delta_{i,j}$ defined as follows.

- When p_j is such that $t < k$, for i from 1 to m, $\delta_{i,j} = \Delta_{i,j}$, i.e. +1 if $h_{i,j} = 0$ and -1 if $h_{i,j} = 1$.
- When p_j is such that $t \geq k$, for i from 1 to m, $\delta_{i,j} = 0$, which means that $\Delta_{1,j}$ to $\Delta_{m,j}$ may be selected according to the second principle.

A first procedure ascertains that at least one set $\{\delta_{i,1} \ldots \delta_{i,f}\}$ is variable or zero, i.e. that at least one number q_1 to q_m is non-trivial or may be chosen as non-trivial.

- For i from 1 to m and j from 1 to f,
- if $\delta_{i,j} = 0$ or $\neq \delta_{i,1}$, return "success".
- Return "failure".

A second procedure ascertains that each set $\{\delta_{i,1} \ldots \delta_{i,f}\}$ is variable or zero, i.e. that each number q_1 to q_m is non-trivial or may be chosen as non-trivial.

- For i from 1 to m,
- For j from 1 to f,

- if $\delta_{i,j} = 0$ or $\neq \delta_{i,1}$, skip to the next value of i,
- Return "failure".
- Return "success".

A third procedure ascertains that each pair of prime factors p_{j1} and p_{j2}, with $1 \leq j_1 \leq j_2 \leq f$, there is at least one set $\{\delta_{i,1} \ldots \delta_{i,f}\}$ where $\delta_{i,j1}$ is zero or different from $\delta_{i,j2}$. It obviously fails when m is smaller than f-1. When it succeeds, among the m numbers q_1 to q_m, there is at least one set of independent f-1 numbers with respect to the f prime factors.

- For j_1 from 1 to f-1 and for j_2 from j_1+1 to f,
- For i from 1 to m,
- If $\delta_{i,j1} = 0$ or $\neq \delta_{i,j2}$, skip to the next values of j_1 and j_2,
- Return "failure".
- Return "success".

When a procedure fails, the generator of GQ2 key sets follows its strategy chosen from the two possible strategies:

- Change one of the m base numbers while keeping the f prime factors,
- Change one of the f prime factors while keeping the m base numbers.

Third principle - The following procedure determines whether the set of generalized GQ2 keys, either during production or already produced, is

- either a set of elementary GQ2 keys, i.e. that the $2 \times m$ numbers $\pm g_1$ to $\pm g_m$ are all non-quadratic residues,
- or else, a set of complementary GQ2 keys, i.e. that among the $2 \times m$ numbers $\pm g_1$ to $\pm g_m$, there is at least one quadratic residue.

The procedure uses both Legendre's symbols $(g_i \mid p_j)$ and $(-g_i \mid p_j)$ for i from 1 to m and for j from 1 to f.

- For i from 1 to m,
- For j from 1 to f,
- If $(g_i \mid p_j) = -1$, skip to the next value of i.
- Return "set of complementary GQ2 keys".

- For j from 1 to f,
- If $(g_i \mid p_j) = -1$, skip to the next value of i.
- Return "set of complementary GK2 keys".
- Return "set of elementary GK2 keys".

Private components - For an equation of the direct type: $x^V \equiv g_i^2 \pmod{p_j}$ the following computation establishes all the possible values for the private component $Q_{i,j}$. The two simplest and most current cases, i.e. $t = 1$ and $t = 2$, are followed by the more complex case, i.e. $t > 2$.

For $t = 1$, i.e. $p_j \equiv 3 \pmod 4$), the key $<(p_j+1)/4, p_j>$ provides the quadratic square root of any quadratic residue in $CG(p_j)$. From this, a number is derived

$s_j \equiv ((p_j+1)/4)^k \pmod{(p_j-1)/2}$, which gives a key $<s_j, p_j>$ transforming G_i into $w \equiv G_i^{sj} \pmod{p_j}$. $Q_{i,j}$ is equal to w or p_j-w.

For $t = 2$, i.e. $p_j \equiv 5 \pmod 8$), the key $<(p_j+3)/8, p_j>$ provides the square root of odd rank for any element of odd rank in $CG(p_j)$. From this, a number is derived $s_j \equiv ((p_j+3)/8)^k \pmod{(p_j-1)/4}$, which gives a key $<s_j, p_j>$ transforming G_i into $w \equiv G_i^{sj} \pmod{p_j}$. Let us note that $z \equiv 2^{(pj-1)/4}$ (mode p_j) is a square root of -1 because 2 is a non-quadratic residue in CG (p_j). $Q_{i,j}$ is either equal to w or to p_j-w or else to $w' \equiv w \times z \pmod{p_j}$ or p_j-w'.

For $p_j \equiv 2^t+1 \pmod{2^{t+1}}$ with $t > 2$, key $<(p_j-1+2^t)/2^{t+1}$, $p_j>$ provides the square root of odd rank of any element of odd rank. The compatibility test between k, g and p gave the value of h, next that of u.

- When G_i is in a cycle ($u = 0$, regardless of the value of k), a number is established,

$s_j \equiv ((p_j-1+2^t)/2^{t+1})^k \pmod{(p_j-1)/2^t}$. Key $<s_j, p_j>$ transforms G_i into the solution of odd rank $w \equiv G_i^{sj} \pmod{p_j}$. There are solutions of even rank distributed in $\min(k, t)$ consecutive branches related to the cycle, let us say, in α branches. $Q_{i,j}$ is equal to the product of w by any of the 2^αth roots of unity in $CG(p_j)$.

- When G_j is in a suitable position in a branch ($u > 0$, $u + k \leq t$), all the solutions are in the same branch as G_j, a branch related to a cycle by the 2^uth power of the number G_i. A number is established as

$$s_j \equiv ((p_j-1+2^t)/2^{t+1})^{k+u} \pmod{(p_j-1)/2^t}.$$

Key $<s_j, p_j>$ transforms the 2^uth power of G_i into a number of odd rank w. The set of products of w with the primitive 2^{k+u}th roots of unity in $CG(p_j)$ comprises the 2^k values of $Q_{i,j}$.

When p_j is such that $t \geq k$, as number b_j is a primitive 2^tth root of unity in $CG(p_j)$, the 2^{t-u}th power of b_j in $CG(p_j)$ exists; this is a primitive 2^kth root of unity. The value of number $\Delta_{i,j}$ may be switched by multiplying $Q_{i,j}$ by a primitive 2^kth root of unity.

For an equation of the inverse type: $1 \equiv x^v \times g_i^2 \pmod{p_j}$, it is sufficient to replace number s_j with $((p_j-1)/2^t)-s_j$ in the key $<s_j, p_j>$, which amounts to inverting the value of $Q_{i,j}$ in $CG(p_j)$.

Example of a set of keys with two prime factors congruent to 5 (mod 8)

p_1 = E6C83BF428689AF8C35E07EDD06F9B39A659829A58B79CD89 4C435C95F32BF25

p_2 = 11BF8A68A0817BFCC00F15731C8B70CEF9204A34133A0DEF8 62829B2EEA74873D

$n = p_1 \times p_2$ = FFFF8263434F173D0F2E76B32D904F56F4A5A6A50008C43D32 B650E9AB9AAD2EB713CD4F9A97C4DBDA3828A3954F296458 D5F42C0126F5BD6B05478BE0A80ED1

Here are the Legendre symbols of the very first prime numbers.

$(2 | p_1) = -1$; $(3 | p_1) = -1$; $(5 | p_1) = +1$; $(7 | p_1) = -1$;
$(11 | p_1) = +1$; $(13 | p_1) = -1$; $(17 | p_1) = +1$;

In $CG(p_1)$, the rank is odd for -5, -11 and 17.

$(2 | p_2) = -1$; $(3 | p_2) = +1$; $(5 | p_2) = +1$; $(7 | p_2) = +1$;

$(11 \mid p_2) = +1$; $(13 \mid p_2) = -1$; $(17 \mid p_2) = -1$;

In $CG(p_2)$, the rank is odd for 3, -5, 7 and 11.

Carmichael's function is

$\lambda(n) = \mathrm{scm}((p_1-1)/4, (p_2-1)/4)$.

λ(n)=33331A13DA4304A5CFD617BD6F83431164212154 3334F40C3D57A9C8558555D5BDAA2EF6AED17B9E3794F51A6 5A1B37239B18FA9B0F618627D8C7E1D8499C1B

With $k = 9$, the number $\sigma \equiv \lambda(n)-((1+\lambda(n))/2)^9 \pmod{\lambda(n)}$ is used as a private exponent, in order to use the generic equations of the inverse type.

σ = 01E66577BC997CAC273671E187A35EFD25373ABC9FE6770E 7446C0CCEF2C72AF6E89D0BE277CC6165F1007187AC58028BD 2416D4CC 1121E7A7A8B6AE186BB4B0

Numbers 2, 3, 7, 13 and 17 are not suitable as base numbers.

Key $<\sigma,n>$ transforms $g_1 = 5$ into a private number Q_1 which does not shows any factorization. Indeed, in both fields, -5 is on a cycle.

Q_1 = 818C23AF3DE333FAECE88A71C4591A70553F91D6C0DD5538E C0F2AAF909B5BDAD491FD8BF13F18E3DA3774CCE19D0097BC 4BD47C5D6E0E7EBF6D89FE3DC5176C

Key $<\sigma,n>$ transforms $g_2 = 11$ into a private number Q_2 which shows a factorization. Indeed, 11 is not in the same position in both fields:

Q_2 = 25F9AFDF177993BE8652CE6E2C728AF31B6D66154D3935AC53 5196B07C19080DC962E4E86ACF40D01FDC454F2565454F2900 50DA052089EEC96A1B7DEB92CCA7

Key $<\sigma,n>$ transforms $g_3 = 21 = 3\times7$ into a private number Q_3 which shows a factorization.

Q_3 = 78A8A2F30FEB4A5233BC05541AF7B684C2406415EA1DD 67D18A0459A1254121E95D5CAD8A1FE3ECFE0685C96CC7EE86 167D99532B3A96B6BF9D93CAF8D4F6AF0

Key <σ,n> transforms $g_4 = 26 = 2 \times 13$ into a private number Q_4 which shows a factorization.

$Q_4 =$
6F1748A6280A200C38824CA34C939F97DD2941DAD300030E
481B738C62BF8C673731514D1978AF5655FE493D659514A6CE
897AB76C01E50B5488C5DAD12332E5

The private key may further be represented by both prime factors, the parameter of the Chinese remainders and eight private components.

$\alpha \equiv (p_2 (\mathrm{mod}\ p_1))^{-1} (\mathrm{mod}\ p_1) =$
ADE4E77B703F5FDEAC5B9AAE825D
649E06692D15FBF0DF737B115DC4D012FD1D

$Q_{1,1} \equiv Q_1 (\mathrm{mod}\ p_1) =$
7751AEE918A8F5CE44AD73D613A4F465E06C6F
9AF4D229949C74DD6C18D76FAF

$Q_{1,2} \equiv Q_1 (\mathrm{mod}\ p_2) =$
A9EB5FA1B2A981AA64CF88C382923DB64376F
5FD48152C08EEB6114F31B7665F

$Q_{2,1} \equiv Q_2 (\mathrm{mod}\ p_1) =$
D5A7D33C5FB75A033F2F0E8B20274B957FA3
4004ABB2C2AC1CA3F5320C5A9049

$Q_{2,2} \equiv Q_2 (\mathrm{mod}\ p_2) =$
76C9F5EFD066C73A2B5CE9758DB512DFC011
F5B5AF7DA8D39A961CC876F2DD8F

$Q_{3,1} \equiv Q_3 (\mathrm{mod}\ p_1) =$
2FEC0DC2DCA5BA7290B27BC8CC85C938A514
B8F5CFD55820A174FB5E6DF7B883

$Q_{3,2} \equiv Q_3 (\mathrm{mod}\ p_2) =$ 010D488E6B0A38A1CC406CEE0D55DE59
013389D8549DE493413F34604A160C1369

$Q_{4,1} \equiv Q_4 (\mathrm{mod}\ p_1) =$
A2B32026B6F82B6959566FADD9517DB8ED852
4652145EE159DF3DC0C61FE3617

$Q_{4,2} \equiv Q_4 (\mathrm{mod}\ p_2) =$
011A3BB9B607F0BD71BBE25F52B305C22489
9E5F1F8CDC2FE0D8F9FF62B3C9860F

Polymorphism of the private key GQ2 - The different possible representations of the private key GQ2 prove to be equivalent: they all amount to knowing the factorization of the module n which is the actual GK2 private key. The representation of the GQ2 private key has an effect on the progress of the computations within the demonstrating entity but not within the controlling entity. Here are the three main representations which are possible for the GQ2 private key. 1) The conventional representation of GQ private keys consists in storing m private numbers Q_i and the public checking key $<v, n>$; for the GQ2 schemes, this representation is in competition with the following two. 2) The optimum representation in terms of work loads consists in storing the parameter k, the f prime factors p_j, the $m \times f$ private components $Q_{i,j}$ and the f-1 parameters of the Chinese remainders. 3) The optimum representation in terms of private key size consists in storing the parameter k, the m base numbers g_i and the f prime factors p_j, and then in starting each use by establishing either m private numbers Q_i and the module n so that it amounts to the first representation, or $m \times f$ private components $Q_{i,j}$ and the f-1 parameters of the Chinese remainders so that it amounts to the second representation.

Because the security of the dynamic authentication mechanism or of the digital signature is equivalent to knowing a factorization of the module, with the GQ2 schemes, it is not possible to distinguish two entities using the same module simply. Generally, each proving entity has its own GQ2 module. However, GQ2 modules with four prime factors may be specified, two of which are known to an entity and the other two to another one.

Dynamic authentication - The dynamic authentication mechanism is for proving to an entity called a **controller**, the authenticity of another entity called a **demonstrator** as well as the authenticity of a possible associated message M, so that the controller ascertains that it

is actually dealing with the demonstrator and optionally that itself and the demonstrator are speaking of the same message M. The associated message M is optional, which means it may be empty.

The dynamic authentication mechanism is a sequence of four acts: an engagement act, a challenge act, a response act and a checking act. The demonstrator plays the engagement and response acts. The controller plays the challenge and checking acts.

Within the demonstrator, a witness may be isolated, in order to isolate the most sensitive parameters and functions of the demonstrator, i.e. the production of commitments and responses. The witness has the parameter k and the GQ2 private key, i.e. the factorization of the module n according to one of the three representations mentioned above: • the f prime factors and the m base numbers, • the $m \times f$ private components, the f prime factors and the f-1 parameters of the Chinese remainders, • the m private numbers and the module n.

The witness may correspond to a particular embodiment, for example • a chip card connected to a PC forming together the demonstrator or even • programs particularly protected within a PC or even • programs particularly protected within a chip card. The thereby isolated witness is similar to the witness defined hereafter within the signing entity. Upon each execution of the mechanism, the witness produces one or more commitments R, and then as many responses D to as many challenges d. Each set $\{R, d, D\}$ forms **a GQ2 triplet.**

In addition to it comprising the witness, the demonstrator also has a hashing function and a message M, if necessary.

The controller has the module n, for example, from a directory of public keys or even from a certificate of public keys; if necessary, it also has the same hashing function and a message M'. The GQ2 public parameters, i.e. numbers k, m and

g_1 to g_m may be provided to the controller by the demonstrator. The controller is able to reconstruct a commitment R' from any challenge d and from any response D. Parameters k and m inform the controller. Unless indicated otherwise, the m base numbers from g_1 to g_m are the first m prime numbers. Each challenge d should include m elementary challenges referenced from d_1 to d_m: one per base number. Each elementary challenge from d_1 to d_m is a number from 0 to $2^{k-1}-1$ (numbers from $v/2$ to $v-1$ are not used). Typically, each challenge is coded by m times k-1 bits (and not m times k bits). For example, with $k = 5$ and $m = 4$ base numbers, 5, 11, 21 and 26, each challenge includes 16 bits transmitted on four quartets. When the possible $(k-1)\times m$ challenges are equally probable, the number $(k-1)\times m$ determines the security brought by each GQ2 triplet: an impostor who by definition, does not know the factorization of the module n, has one chance of success out of $2^{(k-1)\times m}$, exactly. When $(k-1)\times m$ is from 15 to 20, one triplet is sufficient for reasonably ensuring dynamic authentication. In order to achieve any level of security, triplets may be produced in parallel; they may also be produced in sequence, i.e. repeat the execution of the mechanism.

1) **The act of commitment** comprises the following operations.

When the witness does not use the Chinese remainders, it has the parameter k, the m private numbers from Q_1 to Q_m and module n; it randomly and privately picks one or more random numbers r $(0 < r < n)$; and then by successively squaring them k times (mod n), it transforms each random number r into a commitment R.

$$R \equiv r^v \pmod{n}$$

Here is an example with the previous set of keys without the Chinese remainders.

r =
5E94B894AC24AF843131F437C1B1797EF562CFA53AB8AD426C

1AC016F1C89CFDA13120719477C3E2FB4B4566088E10EF9C010E
8F09C60D981512198126091996

R

6BBF9FFA5D509778D0F93AE074D36A07D95FFC38F70C8D7E33
00EBF234FA0BC20A95152A8FB73DE81FAEE5BF4FD3EB7F5EE3E3
6D7068D083EF7C93F6FDDF673A

When the witness uses the Chinese remainders, it has the parameter k, the first f prime factors from p_1 to p_f, f-1 parameters of Chinese remainders and $m \times f$ private components $Q_{i,j}$; it randomly and privately picks one or more collections of f random numbers: every collection includes one random number r_i per prime factor p_i ($0 < r_i < p_i$); and then by successively squaring it k times (mod p_i), it transforms each random number r_i into a commitment component R_i.

$$R_i \equiv r_i^{\,v} \pmod{p_i}$$

For each collection of f commitment components, the witness establishes a commitment according to the Chinese remainder technique. There are as many commitments as there are random number collections

R = Chinese remainders(R_1, R_2,... R_f)

Here is an example with the previous set of keys and with the Chinese remainders.

r_1 = 5C6D37F0E97083C8D120719475E080BBBF9F7392F11F3E2
44FDF0204E84D8CAE

R_1 = 3DDF516EE3945CB86D20D9C49E0DA4D42281D07A7607
4DD4FE20C5C7C5E205DF66

r_2 = AC8F85034AC78112071947C457225E908E83A2621B0154
ED15DBFCB9A4915AC3

R_2 = 01168CEC0F661EAA15157C2C287C6A5B34EE28F8EB4D8D
340858079BCAE4ECB016

R = Chinese remainders (R_1,R_2) =
0AE51D90CB4FDC3DC757C56E063C9ED86BE153B71FC65F47C1
23C27F082BC3DD15273D4A923804718573F2F05E991487D17
DAE0AAB7DF0D0FFA23E0FE59F95F0

In both cases, the demonstrator transmits to the controller, either all or part of each commitment R, or else a hashing code H obtained by hashing each commitment R and a message M.

2) **The act of challenge** consists in randomly picking one or more challenges d, each consisting of m elementary challenges $d_1, d_2, \dots d_m$; each elementary challenge d_i is one of the numbers from 0 to v/2-1.

$d = d_1\, d_2 \dots d_m$

Here is a challenge for both examples, i.e. with $k = 5$ and $m = 4$.

$d_1 = 1011 = 11 =$ 'B'; $d_2 = 0011 = 3$; $d_3 = 0110 = 6$; $d_4 = 1001 = 9$,

$d = d_1 \| d_2 \| d_3 \| d_4 = 1011001101101001 =$ B3 69

The controller transmits each challenge d to the demonstrator.

3) **The act of response** includes the following operations.

When the witness does not use Chinese remainders, it has the parameter k, the m private numbers from Q_1 to Q_m and the module n; it computes one or more responses D by using each random number r from the commitment act and the private numbers according to the elementary challenges.

$$D \equiv r_i \times Q_i^{d1} \times Q_2^{d2} \times \dots Q_m^{dm} \pmod{p_i}$$

Here is the continuation of the example without the Chinese remainders.

$D =$
027E6E808425BF2B401FD00B15B642B1A8453BE8070D86C0A7
870E6C1940F7A6996C2D871EBE611812532AC5875E0E116CC8
BA648FD8E86BE0B2ABCC3CCBBBE4

When the witness uses Chinese remainders, it has the parameter k, f prime factors from p_1 to p_f, the f-1 Chinese remainder parameters and $m \times f$ private components $Q_{i,j}$; it computes one or more collections of f response components by using each collection of random numbers from the

commitment act: each collection of response components includes one component per prime factor.

$$D \equiv r_i \times Q_{1,i}^{d1} \times Q_{2,i}^{d2} \times \ldots Q_{m,i}^{dm} \pmod{p_i}$$

For each collection of response components, the witness establishes a response according to the Chinese remainder technique. There are as many responses as there are challenges.

D = Chinese remainders($D_1, D_2, \ldots D_f$)

Here is the continuation of the example without the Chinese remainders.

$$D1 = r_i \times Q_{1,1}^{d1} \times Q_{2,1}^{d2} \times Q_{3,1}^{d3} \times Q_{4,1}^{d4} \pmod{p_1}$$

C71F86F6FD8F955E2EE434BFA7706E38E5E715375BC2CD2029A
4BD572A9EDEE6

$$D2 = r_2 \times Q_{1,2}^{d1} \times Q_{2,2}^{d2} \times Q_{3,2}^{d3} \times Q_{4,2}^{d4} \pmod{p_2}$$

0BE022F4A20523F98E9F5DBEC0E10887902F3AA48C864A6C354
693AD0B59D85E

90CE7EA43CB8EA89ABDD0C814FB72ADE74F02FE6F098ABB98C
8577A660B9CFCEAECB93BE1BCC356811BF12DD667E2270134C
9073B9418CA5EBF5191218D3FDB3

In both cases, the demonstrator transmits each response D to the controller.

4) **The act of checking** consists in checking that each triplet $\{R, d, D\}$ satisfies an equation of the following type <u>for a non-zero value.</u>

$$R \times \prod_{i=1}^{m} G_i^{d_i} \equiv D^{2^k} \pmod{n} \quad \text{or else} \quad R \equiv D^{2^k} \times \prod_{i=1}^{m} G_i^{d_i} \pmod{n}$$

or, in restoring each commitment : <u>none of them must be zero.</u>

$$R' \equiv D^{2^k} / \prod_{i=1}^{m} G_i^{d_i} \pmod{n} \quad \text{or else} \quad R' \equiv D^{2^k} \times \prod_{i=1}^{m} G_i^{d_i} \pmod{n}$$

Optionally, the controller then computes a hashing code H' by hashing each restored commitment R' and the message M'. Dynamic authentication is successful when the controller thus retrieves what it has received at the end of the commitment act, i.e., all or part of each commitment R, or the hashing code H.

For example, a sequence of elementary operations transforms response D into a commitment R'. The sequence comprises k squares (mod n) separated by k-1 divisions or multiplications (mode n) by base numbers. For the i-th division or multiplication, which is performed between the i-th square and the i+1-th square, the i-th bit of the elementary challenge d_1, indicates if g_1 must be used, the i-th bit of the elementary challenge d_2 indicates if g_2 must be used, ... up to the i-th bit of the elementary d_m which indicates if g_m must be used.

Here is the end of the example without the Chinese remainders.

D =
027E6E808425BF2B401FD00B15B642B1A8453BE8070D86C0A7
870E6C1940F7A6996C2D871EBE611812532AC5875E0E116CC8
BA648FD8E86BE0B2ABCC3CCBBBE4
Take the square modulo n:
88BA681DD641D37D7A7D9818D0DBEA82174073997C6C32F7
FCAB30380C4C6229B0706D1AF6EBD84617771C31B4243C2F0
376CAF5DCE B644F098FAF3B1EB49B39
Multiply by 5 times 26 = 130, i.e. '82' modulo n:
6ECABA65A91C22431C413E4EC7C7B39FDE14C9782C94FD6FA3
CAAD7AFE192B9440C1113CB8DBC45619595D263C1067D3D0
A840FDE008B415028AB3520A6AD49D
Take the square modulo n:
0236D25049A5217B13818B39AFB009E4D7D52B17486EBF844
D64CF75C4F652031041328B29EBF0829D54E3BD17DAD21817
4A01E6E3AA650C6FD62CC274426607

Multiply by 21, i.e. '15' modulo n :
2E7F40960A8BBF1899A06BBB6970CFC5B47C88E8F115B5DA59
4504A92834BA405559256A705ABAB6E7F6AE82F4F33BF9E912
27F0ACFA4A052C91ABF389725E93
Take the square modulo n:
B802171179648AD687E672D3A32640E2493BA2E82D5DC87D
BA2B2CC0325E7A71C50E8AE02E299EF868DD3FB916EBCBC0C5
569B53D42DAD49C956D8572E1285B0

Multiply by 5 times 11 times 21 = 1155, i.e. '483' modulo n:
330556027631 0DEFEC1337EB5BB5810336FDB28E91B350D485
B09188E0C4F1D67E68E9590DB7F9F39C22BDB4533013625011
248A8DC417C667B419D27CB11F72

Take the square modulo n:
8871C494081ABD1AEB8656C38B9BAAB57DBA72A4BD4EF902
9ECBFFF540E55138C9F22923963151FD0753145DF70CE22E9D0
19990E41DB6104005EEB7B1170559

Multiply by 5 times 11 times 26 = 1430, i.e. '596' modulo n:
2CF5F76EEBF128A0701B56F837FF68F81A6A5D175D0AD67A14
DAEC6FB68C362B1DC0ADD6CFC004FF5EEACDF794563BB09A1
7045ECFFF88F5136C7FBC825BC50C

Take the square modulo n:
6BBF9FFA5D509778D0F93AE074D36A07D95FFC38F70C8D7E33
00EBF234FA0BC20A95152A8FB73DE81FAEE5BF4FD3EB7F5EE3E3
6D7068D083EF7C93F6FDDF673A

The commitment r is retrieved. Authentication is successful.

Here is the end of the example with the Chinese remainders.
D =
90CE7EA43CB8EA89ABDD0C814FB72ADE74F02FE6F098ABB98C
8577A660B9CFCEAECB93BE1BCC356811BF12DD667E2270134C
9073B9418CA5EBF5191218D3FDB3

Take the square modulo n :

770192532E9CED554A8690B88F16D013010C903172B266C11 33B136EBE3EB5F13B170DD41F4ABE14736ADD3A70DFA43121 B6FC5560CDD4B4845395763C792A68

Multiply by 5 times 26 = 130, i.e. '82' modulo n:

6EE9BEF9E52713004971ABB9FBC31145318E2A703C8A2FB3E144 E7786397CD8D1910E70FA86262DB771AD1565303AD6E4CC6E 90AE3646B461D3521420E240FD4

Take the square modulo n:

D9840D9A8E80002C4D0329FF97D7AD163D8FA98F6AF8FE2B21 60B2126CBBDFC734E39F2C9A39983A426486BC477F20ED2CA5 9E664C23CA0E04E84F2F0AD65340

Multiply by 21, i.e. '15' modulo n:

D7DD7516383F78944F2C90116E1BEE0CCDC8D7CEC5D7D1795 ED33BFE8623DB3D2E5B6C5F62A56A2DF4845A94F32BF3CAC36 0C7782B5941924BB4BE91F86BD85F

Take the square modulo n:

DD34020DD0804C0757F29A0CBBD7B46A1BAF949214F74FDFE 021B626ADAFBAB5C3F1602095DA39D70270938AE362F2DAE0 B914855310C75BCA328A4B2643DCCDF

Multiply by 5 times 11 times 21 = 1155, i.e. '483' modulo n:

038EF55B4C826D189C6A48EFDD9DADBD2B63A7D675A0587C 8559618EA2D83DF552D24EAF6BE983FB4AFB3DE7D4D254519 0F1B1F946D327A4E9CA258C73A98F57

Take the square modulo n:

D1232F50E30BC6B7365CC2712E5CAE079E47B971DA03185B33 E918EE6E99252DB3573CC87C604B327E5B20C7AB920FDF142A 8909DBBA1C04A6227FF18241C9FE

Multiply by 5 times 11 times 26 = 1430, i.e. '596' modulo n:

3CC768F12AEDFCD4662892B9174A21D1F0DD9127A54AB63C 984019BED9BF88247EF4CCB56D71E0FA30CFB0FF28B7CE45556 F744C1FD751BFBCA040DC9CBAB744

Take the square modulo n:

0AE51D90CB4FDC3DC757C56E063C9ED86BE153B71FC65F47C1
23C27F082BC3DD15273D4A9238047185 73F2F05E991487D17
DAE0AAB7DF0D0FFA23E0FE59F95F0

The commitment r is retrieved correctly. Authentication is successful.

Digital signature

The digital signing mechanism enables an entity called a signing entity to produce signed messages and an entity called controller to ascertain signed messages. Message M is any binary sequence: it may be empty. The message M is signed by adding a signature appendix to it, which comprises one or more commitments and/or challenges, as well as the corresponding responses.

The controller has the module n, for example, from a directory of public keys or even from a certificate of public keys; it also has the same hashing function. The GQ2 public parameters, i.e. numbers k, m and g_1 to g_m may be given to the controller by the demonstrator, for example by putting them in the signature appendix.

Numbers k and m inform the controller. Each elementary challenge from d_1 to d_m on the one hand is a number from 0 to 2^{k-1}-1 (numbers v/2 to v-1 are not used). Each challenge d on the other hand should include m elementary challenges referenced from d_1 to d_m, as many as there are base numbers. Further, unless indicated otherwise, the m base numbers, from g_1 to g_m are the first m prime numbers. With $(k-1)\times m$ being from 15 to 20, it is possible to sign with four GQ2 triplets produced in parallel; with $(k-1)\times m$ being 60 or more, it is possible to sign with only one GQ2 triplet. For example, with $k = 9$ and $m = 8$, only one GQ2 triplet is sufficient; each challenge includes eight bytes and the base numbers are 2,3,5,7,11,13,17 and 19.

The signing operation is a sequence of three acts: a commitment act, a challenge act and a response act. Each act produces one or more GQ2 triplets each comprising: a

commitment r ($\neq 0$), a challenge d consisting of m elementary challenges referenced by $d_1, d_2, \ldots d_m$ and a response D ($\neq 0$).

The signing entity has a hashing function, the parameter k and the GQ2 private key, i.e., the factorization of the module n according to one of the three representations mentioned above. Within the signing entity, it is possible to isolate a witness which executes the commitment and response acts in order to isolate the most sensitive functions and parameters of the demonstrator. In order to compute commitments and responses, the witness has the parameter k and the GQ2 private key, i.e., the factorization of the module n according to one the three representations mentioned above. The thereby isolated witness is similar to the witness defined within the demonstrator. it may correspond to a particular embodiment, for example, • a chip card connected to a PC forming together the signing entity or even • programs particularly protected within a PC, or even, • programs particularly protected within a chip card.

1) **The act of commitment** comprises the following operations.

When the witness has the m private numbers Q_1 to Q_m and the module n, it randomly and privately picks one or more random numbers r ($0 < r < n$); and then, by k successive squarings (mode n), it transforms each random number r into a commitment R.

$$R \equiv r^v \pmod{n}$$

When the witness has f prime factors from p_1 to p_f and $m \times f$ private components $Q_{1,j}$, it randomly and privately picks one or more collections of f random numbers: each collection includes a random number r_i per prime factor p_i ($0 < r_i < p_1$); and then k successive squarings (mod p_i), it transforms each random number r_i into a commitment component R_i,

$$R_i \equiv r_i^v \pmod{p_i}$$

For each collection of f commitment components, the witness establishes a commitment according to the Chinese

remainder technique. There are as many commitments as there are random number collections.

$$r \equiv \text{Chinese remainders } (R_1, R_2, \ldots R_f)$$

2) **The act of challenge** consists in hashing all commitments r and the message m to be signed in order to obtain a hashing code from which the signing entity forms one or more challenges each comprising m elementary challenges; each elementary challenge is a number from 0 to $v/2$-1; for example, with $k = 9$ and $m = 8$, each challenge includes eight bytes. There are as many challenges as there are commitments. $d = d_1, d_2, \ldots d_m$, extracted from the Hash(M, R) result

3) **The act of response** includes the following operations.

When the witness has m private numbers Q_1, to Q_m, and module n, it calculates one or more responses D by using each random number r of the commitment act and the private numbers according to the elementary challenges.

$$X \equiv Q_1^{d1} \times Q_2^{d2} \times \ldots Q_m^{dm} \pmod n$$

$$D \equiv r \times X \pmod n$$

When the witness has f prime factors from p_1 to p_f and $m \times f$ prime components $Q_{i,j}$, it calculates one or more collections of f response components by using each collection of random numbers from the commitment act; each collection of response components includes one component per prime factor

$$X \equiv Q_{1,i}^{d1} \times Q_{2,i}^{d2} \times \ldots Q_{m,i}^{dm} \pmod{p_i}$$

$$D_i \equiv r_i \times X_i \pmod{p_i}$$

For each collection of response components, the witness establishes a response according to the Chinese remainder technique. There are as many responses as there are challenges.

$$D = \text{Chinese remainders } (D_1, D_2, \ldots D_f)$$

The signing entity signs the message M by adding a signature appendix comprising :

- either each GQ2 triplet, i.e., each commitment R, each challenge d and each response D,
- or each commitment R and each corresponding response D,
- or each challenge d and each corresponding response D.

The running of the verification operation depends on the contents of the signature appendix. Three cases are distinguished.

Should the appendix comprises one or more triplets, the checking operation includes two independent processes for which chronology is indifferent. The controller accepts the signed message if and only if, both following conditions are satisfied.

Firstly, each triplet must be consistent (an appropriate relationship of the following type has to be verified) and acceptable (the comparison has to be done on a non-zero value).

$$R \times \prod_{i=1}^{m} G_i^{d_i} \equiv D^{2^k} \pmod{n} \text{ or else } R \equiv D^{2^k} \times \prod_{i=1}^{m} G_i^{d_i} \pmod{n}$$

For example, the response D is converted by a sequence of elementary operations: k squares (mod n) separated by k-l multiplication or division operations (mod n) by base numbers. For the i-th multiplication or division which is performed between the *i-th* square and the i+ 1st square, the i-th bit of the elementary challenge d_1 indicates whether it is necessary to use g_1, the i-th bit of the elementary challenge d_2 indicates whether it is necessary to use g_2, ... up to the *i-th* bit of the elementary challenge d_m which indicates if it is necessary to use g_m. It is thus necessary to retrieve each commitment R present in the signature appendix.

Furthermore, the triplet or triplets must be linked to the message M. By hashing all the commitments R and the message M, a hashing code is obtained from which each challenge d must be recovered.

$d = d_1\ d_2 \ldots d_m$, identical to those extracted from the result Hash(M, R).

If the appendix has no challenge, the checking operation starts with the reconstitution of one or more challenges d' by hashing all the commitments R and the message M.

$d = d'_1\ d'_2 \ldots\ d'_m$, extracted from the result Hash(M, R).

Then, the controller accepts the signed message if and only if each triplet is consistent (an appropriate relationship of the following type is verified) and acceptable (the comparison is done on a non-zero value).

$$R \times \prod_{i=1}^{m} G_i^{d'_i} \equiv D^{2^k} \pmod{n} \text{ or else } R \equiv D^{2^k} \times \prod_{i=1}^{m} G_i^{d'_i} \pmod{n}$$

Should the appendix comprise no commitment, the checking operation starts with the reconstitution of one or more commitments R' according to one of the following two formulae, namely the one that is appropriate. No re-established commitment should be zero.

$$R' \equiv D^{2^k} / \prod_{i=1}^{m} G_i^{d_i} \pmod{n} \text{ or else } R' \equiv D^{2^k} \times \prod_{i=1}^{m} G_i^{d_i} \pmod{n}$$

Then, the controller must hash all the commitments R' and the message M so as to reconstitute each challenge d.

$d = d_1\ d_2 \ldots\ d_m$, identical to those extracted from the result Hash (M, R').

The controller accepts the signed message if and only if each reconstituted challenge is identical to the corresponding challenge in the appendix.

CLAIMS

1. Method designed to prove to a controller entity,

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity,

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values $Q_1, Q_2, \ldots Q_m$ and public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type

$$G_i . \equiv Q_i^v \text{ . mod } n \text{ or } G_i . \equiv Q_i^v \text{ . mod } n$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1 ;

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_m$ |;

the said $p_1, p_2, \ldots p_m$ prime factors and / or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v \equiv g_i^2 \text{ mod } n \quad (1)$$

can be resolved in x in the ring of integers modulo n ;

Second condition

if $G_i \equiv Q_i^v$ mod n, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i.Q_i^v \equiv 1$ mod n, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial) ;

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad (2)$$

$$x^2 \equiv -g_i \bmod n \quad (3)$$

can be resolved in x in the ring of integers modulo n ;

the said method implements, in the following steps, an entity called a witness having f prime factors pi and/or m numbers of base gi and/or parameters of the Chinese remainders of the prime factors and/or the public modulus *n* and/or the m private values Qi and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i$ mod p_j) of the private values Q_i and of the public exponent v;

- the witness computes commitments R in the ring of integers modulo n; each commitment being computed:

. either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value such that $0 < r < n$,

. or

.. by performing operations of the type

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number P_i such that $0 < r_i < P_i$, each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$

.. then by applying the Chinese remainders method,

- the witness receives one or more challenges d; each challenge d comprising m integers d_i hereinafter called

elementary challenges; the witness, on the basis of each challenge d, computes a response D by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2} \ldots Q_m^{dm} \bmod n$$

. or

.. by performing operations of the type:

$$D_i \equiv r_i.Q_{i,1}^{d1}.Q_{i,2}^{d2} \ldots Q_{i,m}^{dm} \bmod p_i$$

.. then by applying the Chinese remainders method;

the said method being such that there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R, d, D}.

2. Method according to claim 1, designed to prove the authenticity of an entity known as a demonstrator to an entity known as the controller, the said demonstrator entity comprising the witness;

the said demonstrator and controller entities executing the following steps:

. **Step 1: act of commitment R**

- at each call, the witness computes each commitment R by applying the process specified according to claim 1,

- the demonstrator sends the controller all or part of each commitment R,

. **Step 2: act of challenge d**

- the controller, after having received all or part of each commitment R, produces challenges d whose number is equal to the number of commitments R and sends the challenges d to the demonstrator,

. **Step 3: act of response D**

- the witness computes the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

- the demonstrator sends each response D to the controller,

case where the demonstrator has transmitted a part of each commitment R if the demonstrator has transmitted a part of each commitment R, the controller, having the m public values G_1, G_2 ..., G_m, computes a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

the controller ascertains that each reconstructed commitment R' reproduces all or part of each commitment R that has been transmitted to it,

case where the demonstrator has transmitted the totality of each commitment R

if the demonstrator has transmitted the totality of each commitment R, the controller, having the m public values G_1, G_2,... G_m, ascertains that each commitment R satisfies a relationship of the type

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

3. Method according to claim 1, designed to provide proof to an entity, known as the controller entity, of the integrity of a message M associated with an entity called a demonstrator entity, the said demonstrator entity comprising the witness ;

the said demonstrator and controller entities executing the following steps:

. **Step 1: act of commitment R**

- at each call, the witness computes each commitment R by applying the process specified according to claim 1,

. **Step 2: act of challenge d**

- the demonstrator applies a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T,
- the demonstrator sends the token T to the controller,
- the controller, after having received a Token T, produces challenges d equal in number to the number of commitments R and sends the challenges d to the demonstrator,

. **Step 3: act of response D**

- the witness computes the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

- the demonstrator sends each response D to the controller,
- the controller, having the m public values $G_1, G_2, \ldots, G_m$, computes a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

- then the controller applies the hashing function h whose arguments are the message M and all or part of each reconstructed commitment R' to reconstruct the token T',
- then the controller ascertains that the token T' is identical to the token T transmitted.

4. Method according to claim 1, designed to produce the digital signature of a message M by an entity known as the signing entity, the said signing entity comprising the witness;

Signing operation

the said signing entity executes a signing operation in order to obtain a signed message comprising:

- the message M,
- the challenges d and/or the commitments R,

- the responses D;

the said signing entity executes the signing operation by implementing the following steps:

. **Step 1: act of commitment R**

- at each call, the witness computes each commitment R by applying the process specified according to claim 1,

. **Step 2: act of challenge d**

- the signing entity applies a hashing function h whose arguments are the message M and each commitment R to obtain a binary train,

- from this binary train, the signing entity extracts challenges d whose number is equal to the number of commitments R,

. **Step 3: act of response D**

- the witness computes the responses D from the challenges d by applying the process specified according to claim 1.

5. Method according to claim 4, designed to prove the authenticity of the message M by checking the signed message through an entity called a controller;

Checking operation

the said controller entity having the signed message executes a checking operation by proceeding as follows:

. **case where the controller has commitments R, challenges d, responses D,**

if the controller has commitments R, challenges d, responses D,

. . the controller ascertains that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or relationships of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the controller ascertains that the message M, the challenges d and the commitments R satisfy the hashing function

$$d = h\ (\text{message}, R)$$

. case where the controller has challenges d and responses D

if the controller has challenges d and responses D,

. . the controller reconstructs, on the basis of each challenge d and response D, commitments R' satisfying relationships of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or relationships of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the controller ascertains that the message M and the challenges d satisfy the hashing function

$$d = h(\text{message}, R')$$

. case where the controller has commitments R and responses D

if the controller has commitments R and responses D,

. . the controller applies the hashing function and reconstructs d'

$$d' = h\ (\text{message}, R)$$

. . the controller ascertains that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or relationships of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

6. System designed to prove, to a controller server,

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity,

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values Q_1, Q_2, ... Q_m and public values G_1, G_2, ... G_m (m being greater than or equal to 1),

- a public modulus n constituted by the product of f prime factors $p_1, p_2, \dots p_f$ (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type:

$$G_i.Q_i^v \equiv 1. \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \dots g_m$, smaller than the f prime factors $p_1, p_2, \dots p_f$;

the said $p_1, p_2, \dots p_f$ prime factors and/or the said m base numbers $g_1, g_2, \dots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v \equiv g_i^2 \bmod n \quad (1)$$

can be resolved in x in the ring of integers modulo n ;

Second condition

if $G_i \quad Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial).

if $G_i.Q_i^v \quad 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial)

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad (2)$$

$$x^2 \equiv -g_i \bmod n \quad (3)$$

can be resolved in x in the ring of integers modulo n;

the said system comprises a witness device, contained especially in a nomad object which, for example, takes the form of a microprocessor-based bank card,

the witness device comprises

- a memory zone containing the f prime factors p_i and/or the m numbers of bases gi and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

the said witness device also comprises:

- random value production means, hereinafter called random value production means of the witness device,
- computation means, hereinafter called means for the computation of commitments R of the witness device, to compute commitments R in the ring of integers modulo n; each commitment being computed:
  - either by performing operations of the type:

$$R_i \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, and r is such that $0 < r < n$;

  - or by performing operations of the type :

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$ each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$ produced by random value production means, then by applying the Chinese remainders method;

the said witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges;
- computation means, hereinafter called means for the computation of the responses D of the witness device for the computation, on the basis of each challenge d, of a response D,
  - either by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2}. \ldots Q_m^{dm} \bmod n$$

. or by performing operations of the type:

$$D \equiv r.Q_{i,1}^{d1}.Q_{i,2}^{d2}. \dots Q_{i,m}^{dm} \bmod p_i$$

and then by applying the Chinese remainders method.

- transmission means to transmit one or more commitments R and one or more responses D; there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R, d, D}.

7. System according to claim 6, designed to prove the authenticity of an entity called a demonstrator and an entity called a controller,
the said system being such that it comprises:

- a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,

- a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the demonstrator device;

the said system enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation for the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

the demonstrator device also has transmission means, hereinafter called the transmission means of the demonstrator device, to transmit all or part of each commitment R to the controller device through the connection means;

. **Step 2: act of challenge d**

the controller device comprises challenge production means for the production, after receiving all or part of each commitment R, of the challenges d equal in number to the number of commitments R,

the controller device also has transmission means, hereinafter denoted transmission means of the controller, to transmit challenges d to the demonstrator through connection means,

. **Step 3: act of response D**

the means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator transmit each response D to the controller,

the controller device also comprises:

- computation means, hereinafter called the computation means of the controller device,

- comparison means, hereinafter called the comparison means of the controller device,

case where the demonstrator has transmitted a part of each commitment R

if the transmission means of the demonstrator have transmitted a part of each commitment R, the computation means of the controller device, having m public values $G_1, G_2, \ldots, G_m$, compute a reconstructed commitment R', from each

challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

the comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received,

case where the controller has transmitted the totality of each commitment R

if the transmission means of the demonstrator have transmitted the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values G_1, G_2, ..., G_m ascertain that each commitment R satisfies a relationship of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

8. System according lo claim 6, designed to give proof to an entity known as a controller, of the integrity of a message M associated with an entity known as a demonstrator, the said system being such that it comprises

- a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,
- a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially

through a data processing communications network, to the demonstrator device;

the said system enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has transmission means, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

. **Step 2: act of challenge d**

the demonstrator device comprises computation means, hereinafter called the computation means of the demonstrator, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T,

the demonstrator device also has transmission means, hereinafter known as the transmission means of the demonstrator device, to transmit each token T through the connection means to the controller device,

the controller device also has challenge production means for the production, after having received the token T, of the challenges d in a number equal to the number of commitments R,

the controller device also has transmission means, hereinafter called the transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means;

. **Step 3: act of response D**

the means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator transmit each response D to the controller,

the controller device also comprises computation means, hereinafter called the computation means of the controller device, having m public values $G_1, G_2, \ldots, G_m$, in order to firstly compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

then, secondly, compute a token T' by applying the hashing function h having as arguments the message M and all or part or each reconstructed commitment R',

the controller device also has comparison means, hereinafter known as the comparison means of the controller device, to compare the token T' with the received token T.

9. System according to claim 6, designed to produce the digital signature of a message M, hereinafter known as the signed message, by an entity called a signing entity;

the signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D;

Signing operation

the said system being such that it comprises a signing device associated with the signing entity, the said signing device being interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object, for example the form of *a* microprocessor in a microprocessor-based bank card,

the said system enabling the execution or the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the signing device through the interconnection means,

. **Step 2: act of challenge d**

the signing device comprises computation means, hereinafter called the computation means of the signing device, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute a binary train and extract, from this binary train, challenges d whose number is equal to the number of commitments R,

. **Step 3: act of response D**

the means for the reception of the challenges d, receive each challenge d coming from the signing device through the interconnection means,

the means for computing the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

the witness device comprises transmission means, hereinafter called means of transmission of the witness device, to transmit the responses D to the signing device through the interconnection means.

10. System according to claim 9, designed to prove the authenticity of the message M by checking the signed message by means of an entity called the controller;

Checking operation

the said system being such that it comprises a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote

server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the signing device;

the said signing device associated with the signing entity comprises transmission means, hereinafter known as the transmission means of the signing device, for the transmission, to the controller device, of the signed message through the connection means, in such a way that the controller device has a signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D;

the controller device comprises:

- computation means hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device.

. **case where the controller device has commitments R, challenges d, responses D**

if the controller has commitments R, challenges d, responses D,

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M, the challenges d and the commitments R satisfy the hashing function:

$$d = h\ (\text{message}, R)$$

. **case where the controller device has challenges d and responses D**

if the controller device has challenges d and responses D,

. . the computation means of the controller device, on the basis of each challenge d and each response D, compute commitments R' satisfying relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function:

$$d = h \text{ (message, R')}$$

case where the controller device has commitments R and responses D

if the controller device has commitments R and responses D,

. . the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h \text{ (message, R)}$$

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

11. Terminal device associated with an entity, taking the form especially of a nomad object, for example the form of a microprocessor in a microprocessor-based bank card, designed to prove to a controller device:

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity;

by means of all or part of the following parameters or derivatives of these parameters:

m pairs of private values $Q_1, Q_2, \ldots Q_m$ and public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),

- a public modulus n constituted by the product of f prime factors $p_1, p_2, \dots p_f$ (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type

$$G_i.Q_i^v \equiv 1 \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1:

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \dots g_m$, smaller than the f prime factors $p_1, p_2, \dots p_f$:

the said $p_1, p_2, \dots p_f$ prime factors and / or the said m base numbers $g_1, g_2, \dots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v \quad g_i^2 \bmod n \quad (1)$$

can be resolved in x in the ring of integers modulo n

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i.Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial) ;

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad (2)$$

$$x^2 \equiv -g_i \bmod n \quad (3)$$

can be resolved in x in the ring of integers modulo n.

the said terminal device comprises a witness device comprising

- a memory zone containing the f prime factors p_i and/or the m numbers of bases g_i and/or parameters of the Chinese

remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

the said witness device also comprises:

- random value production means, hereinafter called random value production means of the witness device,

- computation means, hereinafter called means for the computation of commitments R of the witness device, to compute commitments R in the ring of integers modulo n; each commitment being computed:

• either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, and r is such that $0 < r < n$.

• or by performing operations of the type:

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$ each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$ produced by random value production means, then by applying the Chinese remainders method;

the said witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges;

- computation means, hereinafter called means for the computation of the responses D of the witness device for the computation, on the basis of each challenge d, of a response D,

. either by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2}. \ldots Q_m^{dm} \bmod n$$

. or by performing operations of the type:

$$D \equiv r.Q_{i,1}^{d1}.Q_{i,2}^{d2}. \ldots Q_{i,m}^{dm} \bmod p_i$$

and then by applying the Chinese remainders method.

- transmission means to transmit one or more commitments R and one or more responses D;

there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R, d, D}.

12. Terminal device according to claim 11, designed to prove the authenticity of an entity called a demonstrator to an entity called a controller;

the said terminal device being such that it comprises a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and being capable especially of taking the form of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,

the said demonstrator device also comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server;

the said terminal device enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

- the witness device has transmission means, hereinafter called the transmission means of the witness device to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

the demonstrator device also has transmission means, hereinafter called the transmission means of the

demonstrator, to transmit all or part of each commitment R to the controller device, through the connection means;

. **Steps 2 and 3: act of challenge d, act of response D**

the means of reception or the challenges d of the witness device receive each challenge d coming from the controller device through the connection means between the controller device and the demonstrator device and through the interconnection means between the demonstrator device and the witness device, the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator transmit each response D to the controller that carries out the check.

13. Terminal device according to claim 11, designed to give proof to an entity, known as a controller, of the integrity of a message M associated with an entity known as a demonstrator,

the said terminal device being such that it comprises a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and being capable especially of taking the form of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,

the said demonstrator device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server;

the said terminal device being used to execute the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1;

the witness device has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

. **Steps 2 and 3: act of challenge d, act of response D**

the demonstrator device comprises computation means, hereinafter called the computation means of the demonstrator, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T,

the demonstrator device also has transmission means, hereinafter known as the transmission means of the demonstrator device, to transmit each token T, through the connection means, to the controller device,

(the said controller device produces the same number of challenges d as the number of commitments R, after receiving the token T),

the means of reception of the challenges d of the witness device receive each challenge d coming from the controller device, through the interconnection means, between the controller device and the demonstrator device and through the interconnection means between the demonstrator device and the witness device,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator send each response D to the controller device which carries out the check.

14. Terminal device according to claim 11, designed to produce the digital signature of a message M, hereinafter known as the signed message, by an entity called a signing entity;

the signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D;

the said terminal device being such that it comprises a signing device associated with the signing entity, the said signing device being interconnected with the witness device by interconnection means and possibly taking especially the form of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,

the said signing device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server;

Signing operation

the said terminal device being used to execute the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1, the witness has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the signing device through the interconnection means,

. **Step 2: act of challenge d**

the signing device comprises computation means, hereinafter called the computation means of the signing device, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute a binary train and extract, from this binary train, challenges d whose number is equal to the number of commitments R,

. **step 3: act of response D**

the means for the reception of the challenges d of the witness device receive the challenges d coming from the signing device through the interconnection means, the means for computing the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

the witness device comprises transmission means, hereinafter called means of transmission of the witness device, to transmit the responses D to the signing device, through the interconnection means.

15. Controller device especially taking the form of a terminal or remote server associated with a controller entity, designed to prove:

- the authenticity or an entity and/or
- the integrity of a message M associated with this entity.

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2), unknown to the controller device and the associated controller entity,

the said modulus and the said private and public values being related by relations of the following type

$$G_i . Q_i^v \equiv 1 . \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

where Q_i is a private value, unknown to the controller device, associated with the public value G_i ;

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_f$;

the said $p_1, p_2, \ldots p_f$ prime factors and / or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v \quad g_i^2 \bmod n \quad (1)$$

can be resolved in x in the ring of integers modulo n

Second condition

if $Gi \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial).

if $G_i.Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial) ;

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad (2)$$

$$x^2 \equiv -g_i \bmod n \quad (3)$$

can be resolved in x in the ring of integers modulo n.

16. Controller device according to claim 15, designed to prove the authenticity of an entity called a demonstrator to an entity called a controller;

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a demonstrator device associated with the demonstrator entity;

the said controller device being used to execute the following steps:

. Steps 1 and 2; act of commitment R, act of challenge d

the said controller device also has means for the reception of all or part of the commitments R coming from the demonstrator device through the connection means,

the controller device has challenge production means for the production, after receiving all or part of each commitment R, of the challenges d in a number equal to the number of commitments R, each challenge d comprising m integers di hereinafter called elementary challenges.

the controller device also has transmission means, hereinafter called transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means;

. Steps 3 and 4: act of response, act of checking

the said controller device also comprises:

- means for the reception of the responses D coming from the demonstrator device, through the connection means,
- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device,

case where the demonstrator has transmitted a part of each commitment R.

if the reception means of the demonstrator have received a part of each commitment R, the computation means or the controller device, having m public values $G_1, G_2, \ldots, G_m$ compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type :

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

the comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received,

case where the demonstrator has transmitted the totality of each commitment R

if the reception means of the controller device have received the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values $G_1, G_2, \ldots, G_m$ ascertain that each commitment R satisfies a relationship of the type :

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

17. Controller device according to claim 15, designed to prove the integrity of a message M associated with an entity known as a demonstrator,

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a demonstrator device associated with the demonstrator entity,

the said controller device enabling the execution of the following steps:

. **Steps 1 and 2: act of commitment R, act of challenge d**

the said controller device also has means for the reception of tokens T coming from the demonstrator device through the connection means,

the controller device has challenge production means for the production, after having received the token T, of the challenges d in a number equal to the number of commitments R, each challenge d comprising m integers, hereinafter called elementary challenges ;

the controller device also has transmission means, hereinafter called the transmission means of the controller

device, to transmit the challenges d to the demonstrator through the connection means;

. **Steps 3 and 4: act of response D, act of checking**

the said controller device also comprises:

- means for reception of the responses D coming from the demonstrator device, through the connection means,
- computation means, hereinafter called the computation means of the controller device, having m public values $G_1, G_2, \ldots, G_m$ to firstly compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v / G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

then, secondly, compute a token T' by applying the hashing function h having as arguments the message M and all or part of each reconstructed commitment R', the controller device also comprises

- comparison means, hereinafter caned the comparison means of the controller device, to compare the token T' with the received token T.

18. Controller device according to claim 15, designed to prove the authenticity of the message M by checking a signed message by means of an entity called a signed message;

the signed message sent by a signing device associated with a signing entity having a hashing function h (message, R), comprising:

- the message M,
- the challenges d and/or the commitments R,
- the response D;

Checking operation

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a signing device associated with the signing entity, the said controller device having received the signed message from the signing device, through the connection means,

the controller device comprises:

- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device;

. **case where the controller device has commitments R, challenges d, responses D**

if the controller has commitments R, challenges d, responses D,

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function

$$d = h\ (\text{message}, R)$$

. **case where the controller device has commitments R and responses D**

if the controller device has commitments R and responses D.

. . the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h\ (\text{message}, R')$$

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

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(54) **Title:** METHOD, SYSTEM, DEVICE FOR PROVING AUTHENTICITY OF AN ENTITY OR INTEGRITY OF A MESSAGE

(54) **Titre:** PROCEDE, SYSTEME, DISPOSITIF A PROUVER L'AUTHENTICITE D'UNE ENTITE OU L'INTEGRITE D'UN MESSAGE

(57) **Abstract:** The invention concerns a method whereby the proof is established by: m (≥ 1) pairs of private Q_i and public $G_i = g_i^2$ values; a public module n formed by the product of f($\geq$2) prime factors; an exponent $v = 2^k$ ($k > 1$), linked by the relationships of the type: $G_i . Q_i^v \equiv 1.$ mod n or $G_i \equiv Q_i^v$ mod n. Among the m numbers obtained by increasing Q_i or its inverse modulo n to modulo n square, k-1 times rank, at least one of them is different from $\pm g_i$. Among the 2m equations: $x^2 \equiv g_i$ mod n, $x^2 \equiv -g_i$ mod n, at least one of them has solutions in x in the ring of modulo n integers.

(57) **Abrégé:** La preuve est établie au moyen de: m (≥ 1) couples de valeurs privées Q_i et publiques $G_i = g_i^2$, un module public n constitué par le produit de f (≥ 2) facteurs premiers, un exposant $v = 2^k$ ($k> 1$), liés par des relations du type: $G_i.Q_i^v \equiv 1.$mod n ou $G_i \equiv Q_i^v$ mod n. Parmi les m nombres obtenus en élevant Q_i ou son inverse modulo n au carré modulo n, k-1 fois de rang, au moins l'un d'entre eux est différent de $\pm g_i$. Parmi les 2m équations: $x^2 \equiv g_i$ mod n; $x^2 \equiv - g_i$ mod n, au moins l'une d'entre elles a des solutions en x dans l'anneau des entiers modulo n.

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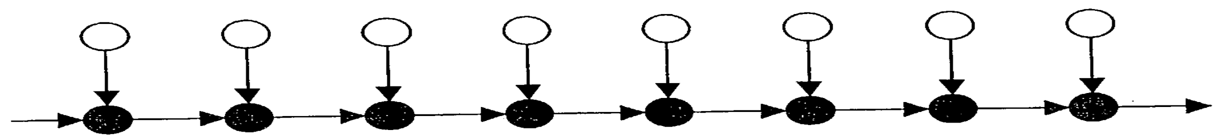

Fig.1A

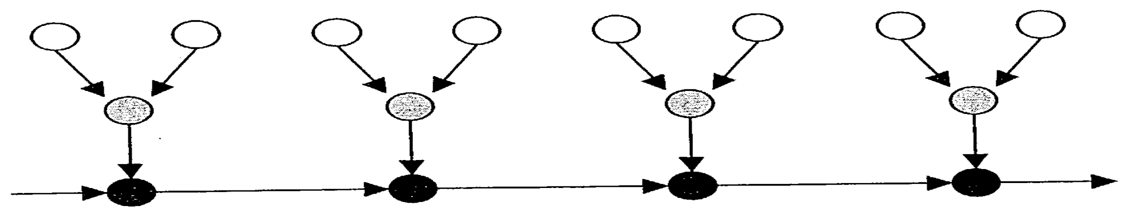

Fig.1B

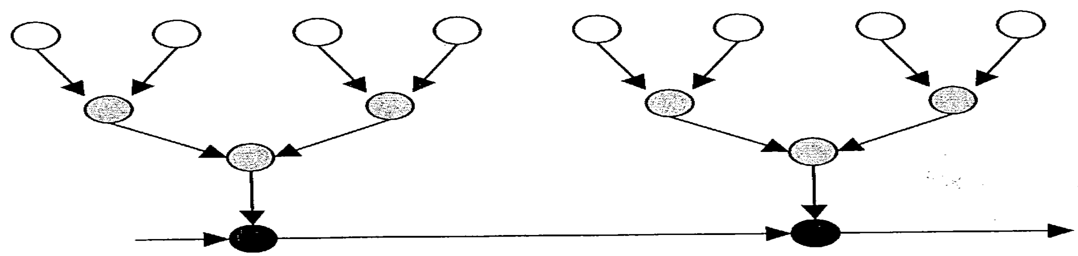

Fig.1C

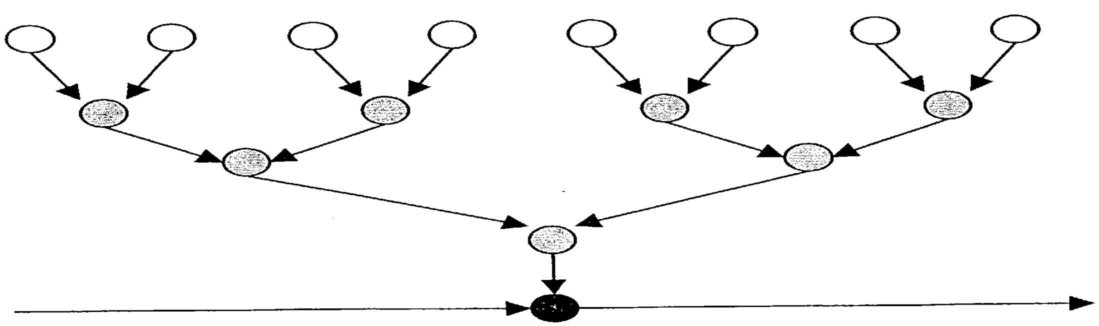

Fig.1D

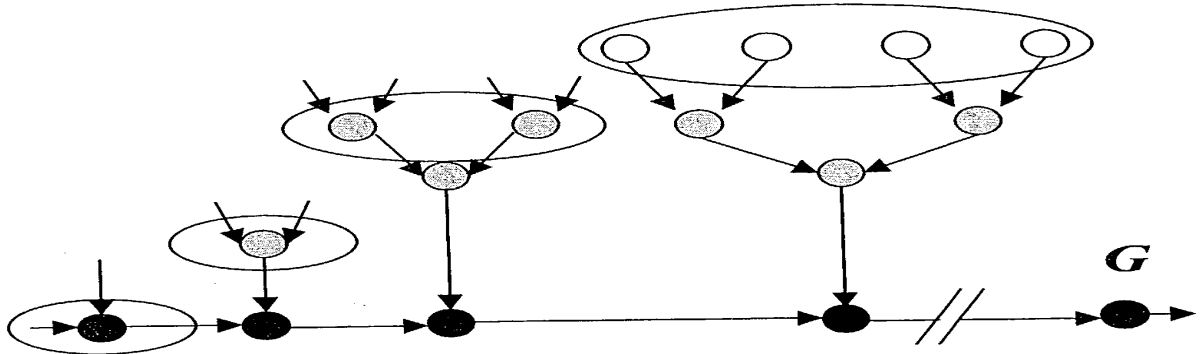

Fig.2A

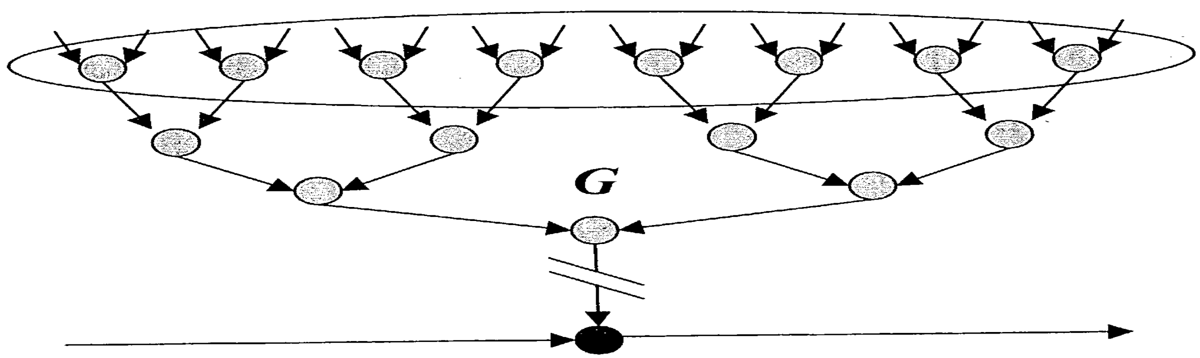

Fig.2B

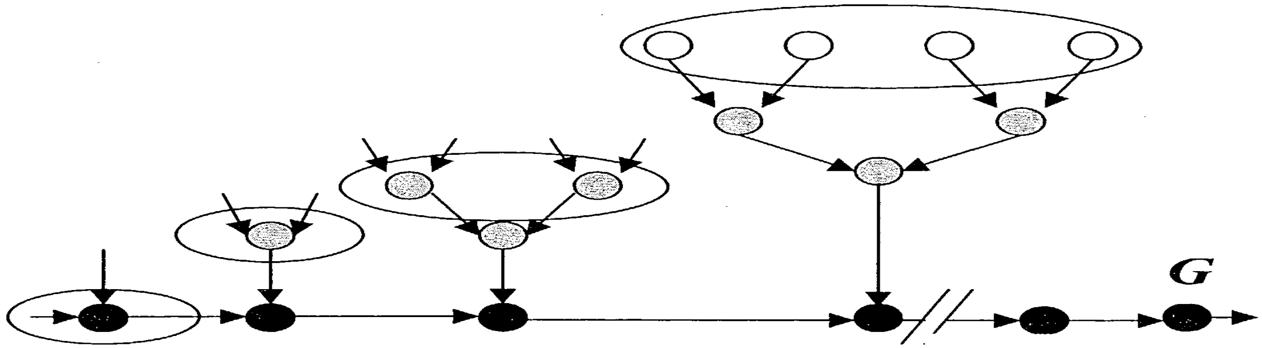

Fig.3A

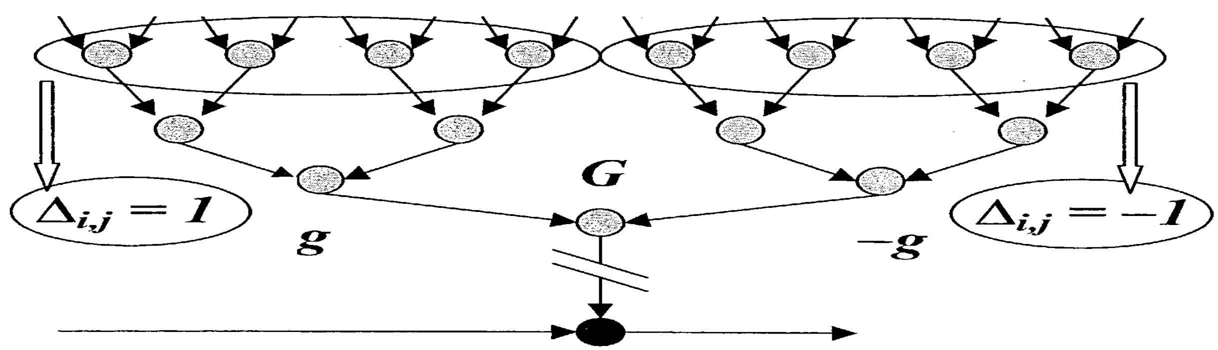

Fig.3B

COMBINED DECLARATION AND POWER OF ATTORNEY IN ORIGINAL APPLICATION

Attorney Docket No.

F40.12-0007

SPECIFICATION AND INVENTORSHIP IDENTIFICATION

As a below named inventor, I declare that:

My residence, post office address and citizenship are as stated below next to my name.

I believe I am the original, first and joint inventor of the subject matter which is claimed, and for which a patent is sought, on the invention entitled METHOD, SYSTEM, DEVICE FOR PROVING AUTHENTICITY OF AN ENTITY OR INTEGRITY OF A MESSAGE the specification of which,

(check one) _X_ is attached hereto.
___ was filed on ______________ as Appln. No. ___________.
___ and was amended on ______________.
_X_ was described and claimed in PCT International Application No. PCT/FR00/02717 filed on 29 September 2000 and as amended under PCT Article 19 on ______________.

ACKNOWLEDGEMENT OF REVIEW OF PAPERS AND DUTY OF CANDOR

I have reviewed and understand the contents of the above identified application, including the claims, as amended by any amendment referred to above. I acknowledge the duty to disclose information which is known to me to be material to the patentability of this application in accordance with 37 C.F.R. § 1.56.

PRIORITY CLAIM (35 U.S.C. § 119)

Prior Foreign Application(s)

I claim foreign priority benefits under 35 U.S.C. § 119(a-d) of any foreign application(s) for patent or inventor's certificate listed below, each of which is incorporated by reference in its entirety, , each of which is incorporated by reference in its entirety, and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on which priority is claimed:

Number	Country	Day/Month/Year Filed	Priority Claimed
FR99 12465	France	1 October 1999	Yes _X_ No ___
FR99 12467	France	1 October 1999	Yes _X_ No ___
FR99 12468	France	1 October 1999	Yes _X_ No ___
FR00 09644	France	21 July 2000	Yes _X_ No ___

Prior Provisional Application(s)

I hereby claim the benefit under 35 U.S.C. §119(e) of any United States Provisional Application(s) listed below, each of which is incorporated by reference in its entirety:

Number	Day/Month/Year Filed

9554 US

PRIORITY CLAIM (35 U.S.C. § 120)

I claim the benefit under 35 U.S.C. § 120 of any United States application(s) listed below, each of which is incorporated by reference in its entirety. Insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of 35 U.S.C. § 112, I acknowledge the duty to disclose to the Patent Office all information known to me to be material to patentability as defined in 37 C.F.R. § 1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

Appln. No.	U.S. Appl. No. (if any under PCT)	Filing Date	Status

DECLARATION

I declare that all statements made herein that are of my own knowledge are true and that all statements that are made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under 18 U.S.C. § 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

POWER OF ATTORNEY

14

I appoint the following attorneys and agents to prosecute the patent application identified above and to transact all business in the Patent and Trademark Office connected therewith, including full power of association, substitution and revocation: Judson K. Champlin, Reg. No. 34,797; Joseph R. Kelly, Reg. No. 34,847; Nickolas E. Westman, Reg. No. 20,147; Steven M. Koehler, Reg. No. 36,188; David D. Brush, Reg. No. 34,557; John D. Veldhuis-Kroeze, Reg. No. 38,354; Deirdre Megley Kvale, Reg. No. 35,612; Theodore M. Magee, Reg. No. 39,758; Christopher R. Christenson, Reg. No 42,413; Brian D. Kaul 41,885; Robert M. Angus, Reg. No. 24,383; Christopher L. Holt, Reg. No. 45,844; Alan G. Rego, Reg. No. 45,956; and David C. Bohn, Reg. No 32,015.

I ratify all prior actions taken by Westman, Champlin & Kelly, P.A. or the attorneys and agents mentioned above in connection with the prosecution of the above-mentioned patent application.

DESIGNATION OF CORRESPONDENCE ADDRESS

Please address all correspondence and telephone calls to **Robert M. Angus** in care of:

WESTMAN, CHAMPLIN & KELLY, P.A.
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1-00

Inventor: L Guillou (Signature) Date: 21 Mai 2002

Inventor: Louis Guillou (Printed Name)

Residence: Bourgbarre, France FRX Citizenship: France

P.O. Address: 16, rue de l'Ise, Bourgbarre, France 35230

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2-00

Inventor: ____________________ Date: 26/04/2002
(Signature)

Inventor: Jean-Jacques Quisquater
(Printed Name)

Residence: Rhode Saint Genese Belgium BEX Citizenship: Belgium

P.O. Address: 3, avenue des Canards, Rhode Saint Genese, Belgium 1640

10089662 .021803

UNITED STATES PATENT AND TRADEMARK OFFICE
DOCUMENT CLASSIFICATION BARCODE SHEET

Miscellaneous

10

Level - 2
Version 1.1
Updated - 8/01/01

10089662 .021803

UNITED STATES PATENT AND TRADEMARK OFFICE
DOCUMENT CLASSIFICATION BARCODE SHEET

371 Application As-Filed

Level - 1
Version 1.1
Updated - 8/01/01

JC05 Rec'd PCT/PTO 29 MAR 2002

TRANSMITTAL LETTER TO THE UNITED STATES DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. § 371		Attorney Docket No. F40.12-0007 U.S. Application No. 10/089662
INTERNATIONAL APPLICATION PCT/FR 00/02717	INTERNATIONAL FILING DATE 29.09.2000	PRIORITY DATE CLAIMED 01.10.99

TITLE OF INVENTION
METHOD, SYSTEM, DEVICE FOR PROVING AUTHENTICITY OF AN ENTITY OR INTEGRITY OF A MESSAGE

APPLICANT(S) FOR DO/EO/US
GIULLOU, Louis et al.

Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. [X] This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. [] This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.
3. [X] This is an express request to begin national examination procedures (35 U.S.C. 371(f). The submission must include items (5), (6), (9) and (20) indicated below.
4. [X] The US has been elected by the expiration of the 19th month from the priority date (Article 31).
5. [X] A copy of the International Application as filed (35 U.S.C. 371(c)(2))
   a. [X] is transmitted herewith (required only if not transmitted by the International Bureau).
   b. [X] has been communicated by the International Bureau.
   c. [] is not required, as the application was filed in the United States Receiving Office (RO/US).
6. [X] A translation of the International Application into English (35 U.S.C. 371(c)(2)).
   a. [X] is attached hereto.
   b. [] has been previously submitted under 35 U.S.C. 154(d)(4).
   c. [] is not required, as the application was filed in English
7. [X] Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371(c)(3))
   a. [] are attached hereto (required only if not transmitted by the International Bureau).
   b. [] have been transmitted by the International Bureau.
   c. [] have not been made; however, the time limit for making such amendments has NOT expired.
   d. [X] have not been made and will not be made.
8. [] A translation of the amendment to the claims under PCT Article 19 (35 U.S.C. 372(c)(3)).
9. [X] An oath unexecuted or declaration of the inventor(s) (35 U.S.C. 371(c)(4)).
10. [] A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 37(c)(5)).

Items 11. to 17. Below concern document(s) or information included:

11. [X] An Information Disclosure Statement under 37 CFR 1.97 and .198.
12. [] An assignment document for recording. A separate cover sheet in compliance with 37 C.F.R. 3.28 and 3.31 is included.
13. [X] A **FIRST** preliminary amendment.
14. [] A **SECOND** or **SUBSEQUENT** preliminary amendment.
15. [] A substitute specification.
16. [] A change of power of attorney and/or address letter.
17. [] A second copy of the published international application under 35 U.S.C. 154(d)(4).
18. [] A second copy of the English language translation of the international application under 35 U.S.C. 154(d)(4).
19. [X] Other items or information: a. [X] Three (3) sheets of drawings.
   b. [X] Abstract typed on a separate page.
   c. [X] File data sheet.

JC13 Rec'd PCT/PTO 29 MAR 2002

U.S. APPLICATION NO.	INTERNATIONAL APPLICATION NO.	ATTORNEY'S DOCKET NUMBER
10/089662	PCT/FR00/02717	F40.12-0007

				CALCULATIONS PTO USE ONLY	
20. [X] The following fees are submitted: **BASIC NATIONAL FEE (37 CFR 1.492(A)(1)-(5)):** Search Report has been prepared by the EPO or JPO....................$860.00 International preliminary examination fee paid to USPTO (37 CFR 1.482)....................$690.00 No international preliminary examination fee paid to USPTO (37 CFR 1.482) but international search fee paid to USPTO (37 CFR 1.445(a)(2))....................$710.00 Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(2)) paid to USPTO....................$1000.00 International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(2)-(4)....................$ 100.00					
ENTER APPROPRIATE BASIC FEE AMOUNT =				$860	
Surcharge of **$130.00** for furnishing the oath or declaration later than [] 20 [] 30 months from the earliest claimed priority date (37 CFR 1.492(e)).				$0	
CLAIMS	**NUMBER FILED**	**NUMBER EXTRA**	**RATE**		
Total claims	18 - 20 =	0	X 18	$0	
Independent claims	4- 3 =	1	X 80	$80	
MULTIPLE DEPENDENT CLAIM (S) (if applicable)			+ $270.00	$0	
TOTAL OF ABOVE CALCULATIONS =				$0	
[] Applicant claims small entity status. See 37 CFR 1.27. The fees indicated above are reduced by 1/2.				$0	
SUBTOTAL =				$940	
Processing fee of $130.00 for furnishing the English translation later than [] 20 [] 30 months from the earliest claimed priority date (37 CFR 1.492(f))				$0	
TOTAL NATIONAL FEE =				$0	
Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31). **$40.00** per property. +				$0	
TOTAL FEES ENCLOSED =				$940	
				Amount to be: refunded	$
				charged	$

a. [X] A check in the amount of $940.00 to cover the above fees is enclosed.

b. [] Please charge my Deposit Account No. **23-1123** in the amount of $ to cover the above fees. A duplicate copy of this sheet is enclosed.

c. [X] The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment, to Deposit Account No. **23-1123**. A duplicate copy of this sheet is enclosed.

NOTE: Where an appropriate time limit under 37 C.F.R. 1.494 or 1.495 has not been met, a petition to revive (1.37(a) or (b)) must be filed and granted to restore the application to pending status.

Send all correspondence to:

Robert M. Angus
WESTMAN, CHAMPLIN & KELLY, P.A.
Suite 1600 - International Centre
900 Second Avenue South
Minneapolis, MN 55402-3319

Robert M. Angus
Signature
Robert M. Angus
Reg. No.

10/089662
JC13 Rec'd PCT/PTO 29 MAR 2002

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

First Named Inventor	:	Louis Guillou et al.	
Appln. No.:			
Filed	:	HEREWITH	Group Art Unit:
For	:	METHOD, SYSTEM, DEVICE FOR PROVING AUTHENTICITY OF AN ENTITY OR INTEGRITY OF A MESSAGE	Examiner:
Docket No.:		F40.12-0007	

PRELIMINARY AMENDMENT

EXPRESS MAIL NO. EV049900733US
DATE OF DEPOSIT: March 29, 2002

Box Non-Fee Amendment
Commissioner for Patents
Washington, D.C. 20231
Sir:

Please amend the above-identified application as follows:

IN THE SPECIFICATION

On Page 1, before line 1 and after the title, please insert the following:

CROSS-REFERENCE TO RELATED APPLICATION

This Application is a Section 371 National Stage of International Application No. PCT/FR00/02717 filed September 29, 2000, published April 12, 2001 as WO 01/26279, not in English.

FIELD OF THE INVENTION

On Page 1, between lines 3 and 4, please insert the following:

BACKGROUND OF THE INVENTION

On Page 3, between lines 8 and 9, please insert the following:

SUMMARY OF THE INVENTION

On Page 33, please delete line 12 and replace with the following:

BRIEF DESCRIPTION OF THE DRAWINGS

FIGS. 1A-1D, 2A, 2B, 3A and 3B are graphs useful in explaining the present invention.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS

IN THE CLAIMS

1. (Amended) Method designed to prove to a controller entity,

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity,

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values Q_1, Q_2, ... Q_m and public values G_1, G_2, ... G_m (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors p_1, p_2, ... p_f (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type

$$G_i . Q_i^v \equiv 1 . \bmod n \text{ or } G_i . \equiv Q_i^v . \bmod n$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_m$|;

the said $p_1, p_2, \ldots p_m$ prime factors and/or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x^v \equiv g_i^2 \bmod n$$

can be resolved in x in the ring of integers modulo n;

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i.Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n$$

$$x^2 \equiv -g_i \bmod n$$

can be resolved in x in the ring of integers modulo n;

the said method implements, in the following steps, an entity called a witness having f prime factors pi and/or m numbers of base gi and/or parameters of the Chinese remainders of the prime factors and/or the public modulus *n* and/or the m private values Qi and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

- the witness computes commitments R in the ring of integers modulo n; each commitment being computed:

. either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value such that $0 < r < n$,

. or

.. by performing operations of the type

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number P_i such that $0<r_i<P_i$, each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$

.. then by applying the Chinese remainders method,

- the witness receives one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges; the witness, on the basis of each challenge d, computes a response D by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2} \ldots Q_m^{dm} \bmod n$$

. or

.. by performing operations of the type:

$$D_i \equiv r_i.Q_{i,1}^{d1}.Q_{i,2}^{d2} \ldots Q_{i,m}^{dm} \bmod p_i$$

.. then by applying the Chinese remainders method;

the said method being such that there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R,d,D}.

2. (Amended) Method according to claim 1, designed to prove the authenticity of an entity known as a demonstrator to an entity known as the controller, the said demonstrator entity comprising the witness;

the said demonstrator and controller entities executing the following steps:

. **Step 1: act of commitment R**

- at each call, the witness computes each commitment R by applying the process specified according to claim 1,

- the demonstrator sends the controller all or part of each commitment R,

. **Step 2: act of challenge d**

- the controller, after having received all or part of each commitment R, produces challenges d whose number is equal to the number of commitments R and sends the challenges d to the demonstrator,

. **Step 3: act of response D**

- the witness computes the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

- the demonstrator sends each response D to the controller,

case where the demonstrator has transmitted a part of each commitment R if the demonstrator has transmitted a part of each commitment R, the controller, having the m public values G_1, G_2 . . ., G_m, computes a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

the controller ascertains that each reconstructed commitment R' reproduces all or part of each commitment R that has been transmitted to it,

case where the demonstrator has transmitted the totality of each commitment R

if the demonstrator has transmitted the totality of each commitment R, the controller, having the m public values G_1, G_2,… G_m, ascertains that each commitment R satisfies a relationship of the type

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n.$$

Claims 3 and 4 are not changed.

5. (Amended) Method according to claim 4, designed to prove the authenticity of the message M by checking the signed message through an entity called a controller;

Checking operation

the said controller entity having the signed message executes a checking operation by proceeding as follows:

. **case where the controller has commitments R, challenges d, responses D,**

if the controller has commitments R, challenges d, responses D,

. . the controller ascertains that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or relationships of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the controller ascertains that the message M, the challenges d and the commitments R satisfy the hashing function

$$d = h\ (\text{message},\ R)$$

. **case where the controller has challenges d and responses D**

if the controller has challenges d and responses D,

. . the controller reconstructs, on the basis of each challenge d and response D, commitments R' satisfying relationships of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or relationships of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the controller ascertains that the message M and the challenges d satisfy the hashing function

$$d=h(\text{message},R')$$

. case where the controller has commitments R and responses D

if the controller has commitments R and responses D,

. . the controller applies the hashing function and reconstructs d'

$$d' = h\ (\text{message},\ R)$$

. . the controller ascertains that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}.\ \dots\ G_m^{dm}.\ D^v \bmod n$$

or relationships of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}.\ \dots\ G_m^{dm}.\ \bmod n.$$

6. (Amended) System designed to prove, to a controller server,

- the authenticity of an entity and/or

- the integrity of a message M associated with this entity,

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values Q_1, Q_2, ... Q_m and public values G_1,G_2, ... G_m (m being greater than or equal to 1),

- a public modulus n constituted by the product of f prime factors p_1, p_2, ... p_f (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type:

$$G_i . Q_i^v \equiv 1. \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1; the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_f$;

the said $p_1, p_2, \ldots p_f$ prime factors and/or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x^v \equiv g_i^2 \bmod n$$

can be resolved in x in the ring of integers modulo n;

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, $k-1$ times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i . Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, $k-1$ times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the $2m$ equations

$$x^2 \equiv g_i \bmod n$$

$$x^2 \equiv -g_i \bmod n$$

can be resolved in x in the ring of integers modulo n;

the said system comprises a witness device, contained especially in a nomad object which, for example, takes the form of a microprocessor-based bank card,

the witness device comprises

- a memory zone containing the f prime factors p_i and/or the m numbers of bases g_i and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

the said witness device also comprises:

- random value production means, hereinafter called random value production means of the witness device,

- computation means, hereinafter called means for the computation of commitments R of the witness device, to compute commitments R in the ring of integers modulo n; each commitment being computed:

• either by performing operations of the type:

$$R_i \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, and r is such that $0 < r < n$;

• or by performing operations of the type:

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$ each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$

produced by random value production means, then by applying the Chinese remainders method;

the said witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges;

- computation means, hereinafter called means for the computation of the responses D of the witness device for the computation, on the basis of each challenge d, of a response D,

. either by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2}. \dots Q_m^{dm} \bmod n$$

. or by performing operations of the type:

$$D \equiv r.Q_{i,1}^{d1}.Q_{i,2}^{d2}. \dots Q_{i,m}^{dm} \bmod p_i$$

and then by applying the Chinese remainders method,

- transmission means to transmit one or more commitments R and one or more responses D;

there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R,d,D}.

7. (Amended) System according to claim 6, designed to prove the authenticity of an entity called a demonstrator and an entity called a controller,

the said system being such that it comprises:

- a demonstrator device associated with the demonstrator entity, the said demonstrator device being

interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,

- a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the demonstrator device;

the said system enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation for the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

the demonstrator device also has transmission means, hereinafter called the transmission means of the demonstrator device, to transmit all or part of each commitment R to the controller device through the connection means;

. **Step 2: act of challenge d**

the controller device comprises challenge production means for the production, after receiving all or part of each commitment R, of the challenges d equal in number to the number of commitments R,

the controller device also has transmission means, hereinafter denoted transmission means of the controller, to transmit challenges d to the demonstrator through connection means,

. **Step 3: act of response D**

the means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator transmit each response D to the controller,

the controller device also comprises:

- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device,

case where the demonstrator has transmitted a part of each commitment R

if the transmission means of the demonstrator have transmitted a part of each commitment R, the computation means of the controller device, having m public values $G_1, G_2, \ldots, G_m$, compute a reconstructed commitment R', from each challenge d and each response

D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n$$

the comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received,

case where the controller has transmitted the totality of each commitment R

if the transmission means of the demonstrator have transmitted the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values G_1, G_2, ..., G_m ascertain that each commitment R satisfies a relationship of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n.$$

8. (Amended) System according to claim 6, designed to give proof to an entity known as a controller, of the integrity of a message M associated with an entity known as a demonstrator,

the said system being such that it comprises

- a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object,

for example the form of a microprocessor in a microprocessor-based bank card,

- a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data processing communications network, to the demonstrator device;

the said system enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has transmission means, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

. **Step 2: act of challenge d**

the demonstrator device comprises computation means, hereinafter called the computation means of the demonstrator, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T,

the demonstrator device also has transmission means, hereinafter known as the transmission means of

the demonstrator device, to transmit each token T through the connection means to the controller device,
the controller device also has challenge production means for the production, after having received the token T, of the challenges d in a number equal to the number of commitments R,

the controller device also has transmission means, hereinafter called the transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means;

. **Step 3: act of response D**

the means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator transmit each response D to the controller,

the controller device also comprises computation means, hereinafter called the computation means of the controller device, having m public values G_1, G_2, . . ., G_m, in order to firstly compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \dots G_m^{dm} . D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v / G_1^{d1} . G_2^{d2} . \dots G_m^{dm} . \bmod n$$

then, secondly, compute a token T' by applying the hashing function h having as arguments the message M and all or part or each reconstructed commitment R',

the controller device also has comparison means, hereinafter known as the comparison means of the controller device, to compare the token T' with the received token T.

Claim 9 is not changed.

10. (Amended) System according to claim 9, designed to prove the authenticity of the message M by checking the signed message by means of an entity called the controller;

Checking operation

the said system being such that it comprises a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the signing device;

the said signing device associated with the signing entity comprises transmission means, hereinafter known as the transmission means of the signing device, for the transmission, to the controller device, of the signed message through the connection means, in such a way that the controller device has a signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D;

the controller device comprises:

- computation means hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device;

. **case where the controller device has commitments R, challenges d, responses D**

if the controller has commitments R, challenges d, responses D,

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M, the challenges d and the commitments R satisfy the hashing function:

$$d = h\ (\text{message}, R)$$

. **case where the controller device has challenges d and responses D**

if the controller device has challenges d and responses D,

. . the computation means of the controller device, on the basis of each challenge d and each

response D, compute commitments R' satisfying relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function:

$$d = h\ (\text{message}, R')$$

case where the controller device has commitments R and responses D

if the controller device has commitments R and responses D,

. . the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h\ (\text{message}, R)$$

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n.$$

11. (Amended) Terminal device associated with an entity, taking the form especially of a nomad object, for example the form of a microprocessor in a microprocessor-based bank card, designed to prove to a controller device:

- the authenticity of an entity and/or

– the integrity of a message M associated with this entity;

by means of all or part of the following parameters or derivatives of these parameters:

m pairs of private values Q_1, Q_2, ... Q_m and public values G_1, G_2, ... G_m (m being greater than or equal to 1),

– a public modulus n constituted by the product of f prime factors p_1, p_2, ... p_f (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type

$$Gi.Q_i^v \equiv 1 \bmod n \text{ or } Gi \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1:

the said m public values G_i being squares g_i^2 of m distinct base numbers g_1, g_2, ... g_m, smaller than the f prime factors p_1, p_2, ... p_f;

the said p_1, p_2, ... p_f prime factors and/or the said m base numbers g_1, g_2, ... g_m being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x^v \equiv g_i^2 \bmod n$$

can be resolved in x in the ring of integers modulo n

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i . Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n$$

$$x^2 \equiv -g_i \bmod n$$

can be resolved in x in the ring of integers modulo n;

the said terminal device comprises a witness device comprising

- a memory zone containing the f prime factors p_i and/or the m numbers of bases g_i and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

the said witness device also comprises:

- random value production means, hereinafter called random value production means of the witness device,

- computation means, hereinafter called means for the computation of commitments R of the witness device,

to compute commitments R in the ring of integers modulo n; each commitment being computed:

- either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, and r is such that $0 < r < n$.

- or by performing operations of the type:

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$ each r_i belonging to a collection of random values $\{r_1, r_2, ... r_f\}$ produced by random value production means, then by applying the Chinese remainders method;

the said witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges;

- computation means, hereinafter called means for the computation of the responses D of the witness device for the computation, on the basis of each challenge d, of a response D,

. either by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2}. \dots Q_m^{dm} \bmod n$$

. or by performing operations of the type:

$$D \equiv r.Q_{i,1}^{d1}.Q_{i,2}^{d2}. \dots Q_{i,m}^{dm} \bmod p_i$$

and then by applying the Chinese remainders method.

- transmission means to transmit one or more commitments R and one or more responses D;
there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R, d, D}.

Claims 12-14 are not changed.

15. (Amended) Controller device especially taking the form of a terminal or remote server associated with a controller entity, designed to prove:

- the authenticity or an entity and/or
- the integrity of a message M associated with this entity.

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2), unknown to the controller device and the associated controller entity,

the said modulus and the said private and public values being related by relations of the following type

$$G_i . Q_i^v \equiv 1. \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

where Q_i is a private value, unknown to the controller device, associated with the public value G_i;

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_f$;

the said $p_1, p_2, \ldots p_f$ prime factors and/or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x^v \equiv g_i^2 \bmod n$$

can be resolved in x in the ring of integers modulo n

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i.Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n$$

$$x^2 \equiv -g_i \bmod n$$

can be resolved in x in the ring of integers modulo n.

16. (Amended) Controller device according to claim 15, designed to prove the authenticity of an entity called a demonstrator to an entity called a controller;

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a demonstrator device associated with the demonstrator entity;

the said controller device being used to execute the following steps:

. **Steps 1 and 2; act of commitment R, act of challenge d**

the said controller device also has means for the reception of all or part of the commitments R coming from the demonstrator device through the connection means,

the controller device has challenge production means for the production, after receiving all or part of each commitment R, of the challenges d in a number equal to the number of commitments R, each challenge d comprising m integers di hereinafter called elementary challenges,

the controller device also has transmission means, hereinafter called transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means;

. **Steps 3 and 4: act of response, act of checking**

the said controller device also comprises:

- means for the reception of the responses D coming from the demonstrator device, through the connection means,

- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device,

case where the demonstrator has transmitted a part of each commitment R

if the reception means of the demonstrator have received a part of each commitment R, the computation means or the controller device, having m public values G_1, G_2, ..., G_m compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n$$

the comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received,

case where the demonstrator has transmitted the totality of each commitment R

if the reception means of the controller device have received the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values G_1, G_2, ..., G_m ascertain that each commitment R satisfies a relationship of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n.$$

Claim 17 is not changed.

18. (Amended) Controller device according to claim 15, designed to prove the authenticity of the message M by checking a signed message by means of an entity called a signed message;

the signed message sent by a signing device associated with a signing entity having a hashing function h (message, R), comprising:

- the message M,
- the challenges d and/or the commitments R,
- the response D;

Checking operation

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a signing device associated with the signing entity, the said controller device having received the signed message from the signing device, through the connection means,

the controller device comprises:

- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device;

. case where the controller device has commitments R, challenges d, responses D

if the controller has commitments R, challenges d, responses D,

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function

$$d = h\ (\text{message},\ R)$$

. **case where the controller device has commitments R and responses D**

if the controller device has commitments R and responses D,

. . the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h\ (\text{message},\ R')$$

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n.$$

REMARKS

Favorable action is respectfully requested.

The Director is authorized to charge any fee deficiency required by this paper or credit any overpayment to Deposit Account No. 23-1123.

Respectfully submitted,

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RMA:tas

MARKED UP VERSION OF REPLACEMENT CLAIMS

1. (Amended) Method designed to prove to a controller entity,

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity,

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values Q_1, Q_2, ... Q_m and public values G_1, G_2, ... G_m (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors p_1, p_2, ... p_f (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type

$$G_i . Q_i^v \equiv 1 . \bmod n \text{ or } G_i . \equiv Q_i^v . \bmod n$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

the said m public values G_i being squares g_i^2 of m distinct base numbers g_1, g_2, ... g_m, smaller than the f prime factors p_1, p_2, ... p_m|;

the said p_1, p_2, ... p_m prime factors and/or the said m base numbers g_1, g_2, ... g_m being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v^v \equiv g_i^2 \bmod n \quad \cancel{(1)}$$

can be resolved in x in the ring of integers modulo n;

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, $k-1$ times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i.Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, $k-1$ times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad \cancel{(2)}$$

$$x^2 \equiv -g_i \bmod n \quad \cancel{(3)}$$

can be resolved in x in the ring of integers modulo n;

the said method implements, in the following steps, an entity called a witness having f prime factors p_i and/or m numbers of base g_i and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the $f.m$ components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

- the witness computes commitments R in the ring of integers modulo n; each commitment being computed:

. either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value such that $0 < r < n$,

. or

.. by performing operations of the type

$$R_i \equiv r_i^{v} \bmod p_i$$

where r_i is a random value associated with the prime number P_i such that $0<r_i<P_i$, each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$

.. then by applying the Chinese remainders method,

- the witness receives one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges; the witness, on the basis of each challenge d, computes a response D by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2} \ldots Q_m^{dm} \bmod n$$

. or

.. by performing operations of the type:

$$D_i \equiv r_i.Q_{i,1}^{d1}.Q_{i,2}^{d2} \ldots Q_{i,m}^{dm} \bmod p_i$$

.. then by applying the Chinese remainders method;

the said method being such that there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R,d,D}.

2. (Amended) Method according to claim 1, designed to prove the authenticity of an entity known as a demonstrator to an entity known as the controller, the said demonstrator entity comprising the witness;

the said demonstrator and controller entities executing the following steps:

. **Step 1: act of commitment R**

- at each call, the witness computes each commitment R by applying the process specified according to claim 1,

- the demonstrator sends the controller all or part of each commitment R,

. **Step 2: act of challenge d**

- the controller, after having received all or part of each commitment R, produces challenges d whose number is equal to the number of commitments R and sends the challenges d to the demonstrator,

. **Step 3: act of response D**

- the witness computes the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

- the demonstrator sends each response D to the controller,

case where the demonstrator has transmitted a part of each commitment R if the demonstrator has transmitted a part of each commitment R, the controller, having the m public values $G_1, G_2 \ldots, G_m$, computes a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n$$

the controller ascertains that each reconstructed commitment R' reproduces all or part of each commitment R that has been transmitted to it,

case where the demonstrator has transmitted the totality of each commitment R

if the demonstrator has transmitted the totality of each commitment R, the controller, having the m public values G_1, G_2,... G_m, ascertains that each commitment R satisfies a relationship of the type

$$R' \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n.$$

Claims 3 and 4 are not changed.

5. (Amended) Method according to claim 4, designed to prove the authenticity of the message M by checking the signed message through an entity called a controller;

Checking operation

the said controller entity having the signed message executes a checking operation by proceeding as follows:

. **case where the controller has commitments R, challenges d, responses D,**

if the controller has commitments R, challenges d, responses D,

. . the controller ascertains that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. D^v \bmod n$$

or relationships of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \dots G_m^{dm}. \bmod n$$

. . the controller ascertains that the message M, the challenges d and the commitments R satisfy the hashing function

$$d = h\ (\text{message},\ R)$$

. **case where the controller has challenges d and responses D**

if the controller has challenges d and responses D,

. . the controller reconstructs, on the basis of each challenge d and response D, commitments R' satisfying relationships of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} .\ D^v \bmod n$$

or relationships of the type

$$R' \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n$$

. . the controller ascertains that the message M and the challenges d satisfy the hashing function

$$d = h(\text{message}, R')$$

. **case where the controller has commitments R and responses D**

if the controller has commitments R and responses D,

. . the controller applies the hashing function and reconstructs d'

$$d' = h\ (\text{message},\ R)$$

. . the controller ascertains that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} .\ D^v \bmod n$$

or relationships of the type

$$R \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n.$$

6. (Amended) System designed to prove, to a controller server,

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity,

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values Q_1, Q_2, ... Q_m and public values G_1, G_2, ... G_m (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors p_1, p_2, ... p_f (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type:

$$G_i . Q_i^v \equiv 1. \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

the said m public values G_i being squares g_i^2 of m distinct base numbers g_1, g_2, ... g_m, smaller than the f prime factors p_1, p_2, ... p_f;

the said p_1, p_2, ... p_f prime factors and/or the said m base numbers g_1, g_2, ... g_m being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v^v \equiv g_i^2 \bmod n \quad \cancel{(1)}$$

can be resolved in x in the ring of integers modulo n;

Second condition

if $G_i \equiv Q_i^v$ mod n, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial)~~.~~,

if $G_i . Q_i^v \equiv 1$ mod n, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad \cancel{(2)}$$

$$x^2 \equiv -g_i \bmod n \quad \cancel{(3)}$$

can be resolved in x in the ring of integers modulo n;

the said system comprises a witness device, contained especially in a nomad object which, for example, takes the form of a microprocessor-based bank card,

the witness device comprises

- a memory zone containing the f prime factors p_i and/or the m numbers of bases gi and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i$ mod p_j) of the private values Q_i and of the public exponent v;

the said witness device also comprises:

- random value production means, hereinafter called random value production means of the witness device,

- computation means, hereinafter called means for the computation of commitments R of the witness device, to compute commitments R in the ring of integers modulo n; each commitment being computed:

• either by performing operations of the type:

$$R_i \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, and r is such that $0 < r < n$;

• or by performing operations of the type:

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$ each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$ produced by random value production means, then by applying the Chinese remainders method;

the said witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges;

- computation means, hereinafter called means for the computation of the responses D of the witness device for the computation, on the basis of each challenge d, of a response D,

. either by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2}. \dots Q_m^{dm} \bmod n$$

. or by performing operations of the type:

$$D \equiv r.Q_{i,1}^{d1}.Q_{i,2}^{d2}. \dots Q_{i,m}^{dm} \bmod p_i$$

and then by applying the Chinese remainders method~~;~~,

- transmission means to transmit one or more commitments R and one or more responses D;

there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R,d,D}.

7. (Amended) System according to claim 6, designed to prove the authenticity of an entity called a demonstrator and an entity called a controller,

the said system being such that it comprises:

- a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,

- a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the demonstrator device;

the said system enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation for the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

the demonstrator device also has transmission means, hereinafter called the transmission means of the demonstrator device, to transmit all or part of each commitment R to the controller device through the connection means;

. **Step 2: act of challenge d**

the controller device comprises challenge production means for the production, after receiving all or part of each commitment R, of the challenges d equal in number to the number of commitments R,

the controller device also has transmission means, hereinafter denoted transmission means of the controller, to transmit challenges d to the demonstrator through connection means,

. **Step 3: act of response D**

the means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. Step 4: act of checking

the transmission means of the demonstrator transmit each response D to the controller,

the controller device also comprises:

- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device,

case where the demonstrator has transmitted a part of each commitment R

if the transmission means of the demonstrator have transmitted a part of each commitment R, the computation means of the controller device, having m public values G_1, G_2, ..., G_m, compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n$$

the comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received,

case where the controller has transmitted the totality of each commitment R

if the transmission means of the demonstrator have transmitted the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values $G_1, G_2, \ldots, G_m$ ascertain that each commitment R satisfies a relationship of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n.$$

8. (Amended) System according ~~to~~to claim 6, designed to give proof to an entity known as a controller, of the integrity of a message M associated with an entity known as a demonstrator,
the said system being such that it comprises

- a demonstrator device associated with the demonstrator entity, the said demonstrator device being interconnected with the witness device by interconnection means and possibly taking the form especially of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card,
- a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data processing communications network, to the demonstrator device;

the said system enabling the execution of the following steps:

. **Step 1: act of commitment R**

at each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified according to claim 1,

the witness device has transmission means, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means,

. **Step 2: act of challenge d**

the demonstrator device comprises computation means, hereinafter called the computation means of the demonstrator, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T,

the demonstrator device also has transmission means, hereinafter known as the transmission means of the demonstrator device, to transmit each token T through the connection means to the controller device, the controller device also has challenge production means for the production, after having received the token T, of the challenges d in a number equal to the number of commitments R,

the controller device also has transmission means, hereinafter called the transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means;

. **Step 3: act of response D**

the means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means,

the means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified according to claim 1,

. **Step 4: act of checking**

the transmission means of the demonstrator transmit each response D to the controller,

the controller device also comprises computation means, hereinafter called the computation means of the controller device, having m public values $G_1, G_2, \ldots, G_m$, in order to firstly compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship *of* the type

$$R' \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n$$

then, secondly, compute a token T' by applying the hashing function h having as arguments the message M and all or part or each reconstructed commitment R',

the controller device also has comparison means, hereinafter known as the comparison means of the controller device, to compare the token T' with the received token T.

Claim 9 is not changed.

10. (Amended) System according to claim 9, designed to prove the authenticity of the message M by checking the signed message by means of an entity called the controller;

Checking operation

the said system being such that it comprises a controller device associated with the controller entity, the said controller device especially taking the form of a terminal or remote server, the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the signing device;

the said signing device associated with the signing entity comprises transmission means, hereinafter known as the transmission means of the signing device, for the transmission, to the controller device, of the signed message through the connection means, in such a way that the controller device has a signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D;

the controller device comprises:

- computation means hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device~~.~~;

. case where the controller device has commitments R, challenges d, responses D

if the controller has commitments R, challenges d, responses D,

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M, the challenges d and the commitments R satisfy the hashing function:

$$d = h\ (\text{message}, R)$$

. **case where the controller device has challenges d and responses D**

if the controller device has challenges d and responses D,

. . the computation means of the controller device, on the basis of each challenge d and each response D, compute commitments R' satisfying relationships of the type:

$$R \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function:

$$d = h\ (\text{message}, R')$$

case where the controller device has commitments R and responses D

if the controller device has commitments R and responses D,

. . the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h\ (\text{message}, R)$$

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1} . G_2^{d2} . \dots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v / G_1^{d1} . G_2^{d2} . \dots G_m^{dm} . \bmod n.$$

11. (Amended) Terminal device associated with an entity, taking the form especially of a nomad object, for example the form of a microprocessor in a microprocessor-based bank card, designed to prove to a controller device:

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity;

by means of all or part of the following parameters or derivatives of these parameters:

m pairs of private values Q_1, Q_2, ... Q_m and public values G_1, G_2, ... G_m (m being greater than or equal to 1),

- a public modulus n constituted by the product of f prime factors p_1, p_2, ... p_f (f being greater than or equal to 2),

the said modulus and the said private and public values being related by relations of the following type

$$G_i . Q_i^v \equiv 1 \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1:

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_f$;

the said $p_1, p_2, \ldots p_f$ prime factors and/or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x_v^v \equiv g_i^2 \bmod n \cancel{(1)}$$

can be resolved in x in the ring of integers modulo n

Second condition

if $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial),

if $G_i . Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \cancel{(2)}$$

$$x^2 \equiv -g_i \bmod n \cancel{(3)}$$

can be resolved in x in the ring of integers modulo n~~.~~;

the said terminal device comprises a witness device comprising

- a memory zone containing the f prime factors p_i and/or the m numbers of bases g_i and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v;

the said witness device also comprises:

- random value production means, hereinafter called random value production means of the witness device,

- computation means, hereinafter called means for the computation of commitments R of the witness device, to compute commitments R in the ring of integers modulo n; each commitment being computed:

  - either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, and r is such that $0 < r < n$.

  - or by performing operations of the type:

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$ each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$

produced by random value production means, then by applying the Chinese remainders method;

the said witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges;

- computation means, hereinafter called means for the computation of the responses D of the witness device for the computation, on the basis of each challenge d, of a response D,

. either by performing operations of the type:

$$D \equiv r.Q_1^{d1}.Q_2^{d2}. \dots Q_m^{dm} \bmod n$$

. or by performing operations of the type:

$$D \equiv r.Q_{i,1}^{d1}.Q_{i,2}^{d2}. \dots Q_{i,m}^{dm} \bmod p_i$$

and then by applying the Chinese remainders method.

- transmission means to transmit one or more commitments R and one or more responses D;

there are as many responses D as there are challenges d as there are commitments R, each group of numbers R, d, D forming a triplet referenced {R, d, D}.

Claims 12-14 are not changed.

15. (Amended) Controller device especially taking the form of a terminal or remote server associated with a controller entity, designed to prove:

- the authenticity or an entity and/or

- the integrity of a message M associated with this entity.

by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),

- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2), unknown to the controller device and the associated controller entity,

the said modulus and the said private and public values being related by relations of the following type

$$G_i \,.\, Q_i^v \equiv 1 \,.\, \bmod n \text{ or } G_i \equiv Q_i^v \bmod n;$$

where v denotes a public exponent of the form:

$$v = 2^k$$

where k is a security parameter greater than 1;

where Q_i is a private value, unknown to the controller device, associated with the public value G_i;

the said m public values G_i being squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$, smaller than the f prime factors $p_1, p_2, \ldots p_f$;

the said $p_1, p_2, \ldots p_f$ prime factors and/or the said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied:

First condition

each of the equations:

$$x^v \equiv g_i^2 \bmod n \quad \cancel{(1)}$$

can be resolved in x in the ring of integers modulo n

Second condition

if $Gi \equiv Qi^{v}$ mod n, among the m numbers q_i obtained by taking Q_i squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial)~~.~~,

if $G_i.Q_i^{v} \equiv 1$ mod n, among the m numbers q_i obtained by taking the inverse of Q_i modulo n squared modulo n, k-1 times, one of them is not equal to $\pm g_i$ (in other words is not trivial);

Third condition

at least one of the 2m equations

$$x^2 \equiv g_i \bmod n \quad \cancel{(2)}$$

$$x^2 \equiv -g_i \bmod n \quad \cancel{(3)}$$

can be resolved in x in the ring of integers modulo n.

16. (Amended) Controller device according to claim 15, designed to prove the authenticity of an entity called a demonstrator to an entity called a controller;

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a demonstrator device associated with the demonstrator entity;

the said controller device being used to execute the following steps:

. **Steps 1 and 2; act of commitment R, act of challenge d**

the said controller device also has means for the reception of all or part of the commitments R coming

from the demonstrator device through the connection means,

the controller device has challenge production means for the production, after receiving all or part of each commitment R, of the challenges d in a number equal to the number of commitments R, each challenge d comprising m integers di hereinafter called elementary challenges~~.~~,

the controller device also has transmission means, hereinafter called transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means;

. **Steps 3 and 4: act of response, act of checking**

the said controller device also comprises:

- means for the reception of the responses D coming from the demonstrator device, through the connection means,
- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device,

case where the demonstrator has transmitted a part of each commitment R~~.~~

if the reception means of the demonstrator have received a part of each commitment R, the computation means or the controller device, having m public values $G_1, G_2, \ldots, G_m$ compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

the comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received,

case where the demonstrator has transmitted the totality of each commitment R

if the reception means of the controller device have received the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values G_1, G_2, ..., G_m ascertain that each commitment R satisfies a relationship of the type:

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. D^v \bmod n$$

or a relationship *of* the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod \underline{n.}$$

Claim 17 is not changed.

18. (Amended) Controller device according to claim 15, designed to prove the authenticity of the message M by checking a signed message by means of an entity called a signed message;

the signed message sent by a signing device associated with a signing entity having a hashing function h (message, R), comprising:

- the message M,
- the challenges d and/or the commitments R,
- the response D;

Checking operation

the said controller device comprising connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a signing device associated with the signing entity, the said controller device having received the signed message from the signing device, through the connection means,

the controller device comprises:

- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device;

. case where the controller device has commitments R, challenges d, responses D

if the controller has commitments R, challenges d, responses D,

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}.\ D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1}.G_2^{d2}. \ldots G_m^{dm}. \bmod n$$

. . the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function

$$d = h\ (\text{message},\ R)$$

. case where the controller device has commitments R and responses D

if the controller device has commitments R and responses D,

. . the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h\ (\text{message}, R')$$

. . the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . \bmod n.$$

3/prts

METHOD, SYSTEM, DEVICE FOR PROVING THE AUTHENTICITY OF AN ENTITY OR THE INTEGRITY OF A MESSAGE

The present invention relates to the methods, systems and devices designed to prove the authenticity of an entity and/or the integrity and/or authenticity of a message.

The patent EP 0 311 470 B1, whose inventors are Louis Guillou and Jean-Jacques Quisquater, describes such a method. Hereinafter, reference shall be made to their work by the terms "GQ patent" or "GQ method". Hereinafter, the expression "GQ2", or "GQ2 invention" or "GQ2 technology" shall be used to describe the present invention.

According to the GQ method, an entity known as a "trusted authority" assigns an identity to each entity called a "witness" and computes its RSA signature. In a customizing process, the trusted authority gives the witness an identity and signature. Thereafter, the witness declares the following: "*Here is my identity; I know its RSA signature*". The witness proves that he knows the RSA signature of his identity without revealing it. Through the RSA public identification key distributed by the trusted authority, an entity known as a "controller" ascertains, without obtaining knowledge thereof, that the RSA signature corresponds to the declared identity. The mechanisms using the GQ method run "without transfer of knowledge". According to the GQ method, the witness does not

know the RSA private key with which the trusted authority signs a large number of identities.

The GQ technology described above makes use of RSA technology. However, whereas RSA technology truly depends on the factorization of the modulus n, this dependence is not an equivalence, indeed far from it, as can be seen in what are called "multiplicative attacks" against the various standards of digital signatures implementing RSA technology.

The goal of the GQ2 technology is twofold: on the one hand, to improve the performance characteristics of RSA technology and, on the other hand, to avert the problems inherent in RSA technology. Knowledge of the GQ2 private key is equivalent to knowledge of the factorization of the modulus n. Any attack on the triplets GQ2 leads to factorization of the modulus n: this time there is equivalence. With the GQ2 technology, the work load is reduced both for the signing or self-authenticating entity and for the controller entity. Through a better use of the problem of factorizing in terms of both security and performance, the GQ2 technology averts the drawbacks of RSA technology.

The GQ method implements modulo computations of numbers comprising 512 bits or more. These computations relate to numbers having substantially the same size raised to powers of the order of $2^{16}+1$. But existing microelectronic infrastructures, especially in the field of bank cards, make use of monolithic self-programmable microprocessors without arithmetical coprocessors. The work load related to the multiple arithmetical applications involved in methods such as the GQ method leads to computation times which, in certain cases, prove to be disadvantageous for consumers using bank cards to pay for their purchases. It may be recalled here that, in seeking to increase the security of payment cards, the banking authorities have raised a problem that is particularly difficult to solve. In fact, two apparently contradictory questions have to be examined: on the one hand, increasing

security by using increasingly lengthy and distinct keys for each card while, on the other hand, preventing the work load from leading to excessive computation times for the users. This problem becomes especially acute inasmuch as it is also necessary to take account of the existing infrastructure and the existing microprocessor components.

The GQ2 technology is aimed at providing a solution to this problem while still increasing security.

Method

More particularly, the invention relates to a method designed to prove the following to a controller entity:

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity.

This proof is established by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values $Q_1, Q_2, \ldots Q_m$ and public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2).

Said modulus and said private and public values are related by relationships of the type:

$$G_i \,.\, Q_i^v \equiv 1 \,.\, \text{mod } n \text{ or } G_i \equiv Q_i^v \text{ mod } n$$

where v represents a public exponent of the type

$$v = 2^k$$

where k is a security parameter greater than 1, said m public values G_i being the squares g_i^2 of m distinct base numbers $g_1\ g_2, \ldots g_m$ inferior to the f prime factors $p_1, p_2, \ldots p_f$; said f prime factors $p_1, p_2, \ldots p_f$ and/or said m base numbers $g_1\ g_2, \ldots g_m$ being produced in such a way that the following conditions are satisfied.

First condition :

According to the first condition, each of the equations

$$x^v \equiv g_i^2 \text{ mod } n \quad (1)$$

has solutions in x in the ring of integers modulo n.

Second condition :

According to the second condition, in the case where $G_i \equiv Q_i^v \bmod n$, among the m numbers q_i obtained by raising Q_i to the square modulo n, k-1 rank times, one of them is different from $\pm g_i$ (that is to say non-trivial).

According to the second condition, in the case where $G_i \cdot Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by raising the inverse of Q_i modulo n to the square modulo n, k-1 rank times, one of them is different from $\pm g_i$ (that is to say non-trivial).

It is to be noted here that according to current notation $\pm g_i$ represents the number g_i and $n-g_i$.

Third condition :

According to the third condition, among the 2 m equations:

$$x^2 \equiv g_i \bmod n \quad (2)$$

$$x^2 \equiv -g_i \bmod n \quad (3)$$

at least one of them has solutions in x in the ring of integers modulo n.

The method implements an entity called a witness in the steps defined here below. Said witness entity has f prime factors p_i and/or m base numbers g_i and/or parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or the f.m components $Q_{i,j}$ ($Q_{i,j} \equiv Q_i \bmod p_j$) of the private values Q_i and of the public exponent v.

The witness computes commitments R in the ring of integers modulo n. Each commitment is computed either by:

- performing operations of the type

$$R \equiv r^v \bmod n$$

Where r is a random value such that $0<r<n$,

- or

** by performing operations of the type

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$, each r_i belonging to a collection of random values $\{r_1, r_2, \dots r_f\}$,

** then by applying the Chinese remainder method.

The witness receives one or more challenges d. Each challenge d comprises m integers d_i hereinafter called elementary challenges. The witness, on the basis of each challenge d_i computes a response D,

- either by performing operations of the type:

$$D \equiv r \,.\, Q_1^{d1} \,.\, Q_2^{d2} \,.\, \dots\, Q_m^{dm} \bmod n$$

- or

** by performing operations of the type:

$$D_i \equiv r_i \,.\, Q_{i,1}^{d1},\, Q_{i2}^{d2} \,.\, \dots\, Q_{i,m}^{dm} \bmod p_i$$

** then by applying the Chinese remainder method.

The method is such that there are as many responses D as there are challenges d as there are commitments R. Each group of numbers R, d, D forms a triplet referenced {R, d, D}.

Case of the proof of the authenticity of an entity

In a first variant of an embodiment, the method according to the invention is designed to prove the authenticity of an entity known as a demonstrator to an entity known as the controller. Said demonstrator entity comprises the witness. Said demonstrator and controller entities execute the following steps:

- **Step 1: act of commitment R**

At each call, the witness computes each commitment R by applying the process specified above. The demonstrator sends the controller all or part of each commitment R.

- **Step 2: act of challenge d**

The controller, after having received all or part of each commitment R, produces challenges d equal in number to the number of commitments R and sends the challenges d to the demonstrator.

- **Step 3: act of response D**

The witness computes the responses D from the challenges d by applying the process specified above.

• **Step 4: act of checking**

The demonstrator sends each response D to the controller.

First case : the demonstrator has transmitted a part of each commitment R.

If the demonstrator has transmitted a part of each commitment R, the controller, having the m public values G_1, G_2, ..., G_m, computes a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type

$$R' \equiv G_1^{d1} . G_2^{d2} . ... G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v/G_1^{d1} . G_2^{d2} . ... G_m^{dm} . \bmod n.$$

The controller ascertains that each reconstructed commitment R' reproduces all or part of each commitment R that has been transmitted to it.

Second case : the demonstrator has transmitted the totality of each commitment R

If the demonstrator has transmitted the totality of each commitment R, the controller, having the m public values G_1, G_2, ..., G_m, ascertains that each commitment R satisfies a relationship of the type

$$R \equiv G_1^{d1} . G_2^{d2} . ... G_m^{dm} . D^v \bmod n$$

or a relationship of the type

$$R \equiv D^v/G_1^{d1} . G_2^{d2} . ... G_m^{dm} . \bmod n.$$

Case of the proof of the integrity of the message

In a second variant of an embodiment capable of being combined with the first one, the method according to the invention is designed to provide proof to an entity, known as the controller entity, of the integrity of a message M associated with an entity called a demonstrator entity. Said demonstrator entity comprises the witness.

Said demonstrator and controller entities perform the following steps:

• Step 1: act of commitment R

At each call, the witness computes each commitment R by applying the process specified above.

• Step 2: act of challenge d

The demonstrator applies a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T. The demonstrator sends the token T to the controller. The controller, after having received a token T, produces challenges d equal in number to the number of commitments R and sends the challenges d to the demonstrator.

• Step 3: act of response D

The witness computes the responses D from the challenges d by applying the process specified above.

• Step 4: act of checking

The demonstrator sends each response D to the controller. The controller, having the m public values $G_1, G_2, \ldots G_m$, computes a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \,.\, D^v \bmod n$$

or a relationship of the type

$$R' \equiv D^v / G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \,.\, \bmod n.$$

Then the controller applies the hashing function h whose arguments are the message M and all or part of each reconstructed commitment R' to reconstruct the token T'. Then the controller ascertains that the token T' is identical to the token T transmitted.

Digital signature of a message and proof of its authenticity

In a third variant of an embodiment according to the invention, capable of being combined with the two preceding embodiments, the method according to the invention is

designed to produce the digital signature of a message M by an entity known as the signing entity. Said signing entity includes the witness.

Signing operation

Said signing entity executes a signing operation in order to obtain a signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D.

Said signing entity executes the signing operation by implementing the following steps:

• **Step 1: act of commitment R**

At each call, the witness computes each commitment R by applying the process specified above.

• **Step 2: act of challenge d**

The signing entity applies a hashing function h whose arguments are the message M and each commitment R to obtain a binary train. From this binary train, the signing entity extracts challenges d in a number equal to the number of commitments R.

• **Step 3: act of response D**

The witness computes the responses D from the challenges d by applying the process specified process above.

Checking operation

To prove the authenticity of the message M, an entity called a controller checks the signed message. Said controller entity having the signed message carries out a checking operation by proceeding as follows,

• **Case where the controller has commitments R, challenges d, responses D**

If the controller has commitments R, challenges d, responses D, the controller ascertains that the commitments R, the challenges d and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or relationships of the type:

$$R \equiv D^v/G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \,.\, \bmod n.$$

Then the controller ascertains that the message M, the challenges d and the commitments R satisfy the hashing function:

$$d = h\ (\text{message},\ R)$$

• **Case where the controller has challenges d and responses D**

If the controller has challenges d and responses D, the controller, on the basis of each challenge d and each response D, reconstructs commitments R' satisfying relationships of the type:

$$R' \equiv G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \,.\, D^v \bmod n$$

or relationships of the type:

$$R' \equiv D^v/G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \,.\, \bmod n.$$

Then the controller ascertains that the message M and the challenges d satisfy the hashing function:

$$D = h\ (\text{message},\ R')$$

• **Case where the controller has commitments R and responses D**

If the controller has commitments R and responses D, the controller applies the hashing function and reconstructs d'

$$d' = h\ (\text{message},\ R).$$

Then the controller ascertains that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d'1} \,.\, G_2^{d'2} \,.\, \ldots\, G_m^{d'm} \,.\, D^v \bmod n$$

or relationships of the type:

$$R \equiv D^v/G_1^{d'1} \,.\, G_2^{d'2} \,.\, \ldots\, G_m^{d'm} \,.\, \bmod n.$$

System

The present invention also relates to a system designed to prove the following to a controller server:

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity.

This proof is established by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values $Q_1, Q_2, \ldots Q_m$ and public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2).

Said modulus, and said private and public values are linked by relationships of the type:

$$G_i \,.\, Q_i^v \equiv 1 \,.\, \bmod n \text{ or } G_i \equiv Q_i^v \bmod n$$

v designating a public exponent of the type:

$$v = 2^k$$

where k is a security parameter greater than 1,

said m public values G_i being the squares g_i^2 of distinct m base numbers $g_1, g_2, \ldots g_m$ inferior to the f prime factors $p_1, p_2, \ldots p_f$; said f prime factors $p_1, p_2, \ldots p_f$ and /or said m base numbers $g_1, g_2 \ldots g_m$ being produced such that the following conditions are satisfied.

First condition

According to the first condition, each of the equations:

$$x^v \equiv g_i^2 \,.\, \bmod n \quad (1)$$

can be solved in x in the ring of integers modulo n.

Second condition

According to the second condition, in the case where $G_i \equiv Q_i^v \,.\, \bmod n$, among the m numbers q_i obtained by raising Q_i to the square modulo n, $k-1$ rank times, one of them is different from $\pm g_i$ (that is to say non trivial).

According to the second condition, in the case where $G_i \,.\, Q_i^v \equiv 1 \bmod n$, among the m numbers q_i obtained by raising the inverse of Q_i modulo n to the square modulo n, $k-1$ rank times, one of them is different from $\pm g_i$ (that is to say non trivial).

It is pointed out here that according to a current notation $\pm g_i$ represents the numbers g_i and $n-g_i$.

Third condition:

According to the third condition, among the 2m equations:

$$x^2 \equiv g_i \bmod n \quad (2)$$
$$x^2 \equiv g_i \bmod n \quad (3)$$

at least one of them can be solved in x in the ring of integers modulo n.

Said system comprises a witness device, contained especially in a nomad object which, for example, takes the form of a microprocessor-based bank card. The witness device comprises a memory zone containing the f prime factors p_i and/or the parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or f.m components $Q_{i,j}(Q_{i,j} \equiv Q_i \bmod p_j)$ of the private values Q_i and of the public exponent v. The witness device also comprises:

- random value production means hereinafter called random value production means of the witness device,
- computation means, hereinafter called means for the computation of commitments R of the witness device.

The means of computation make it possible to compute commitments R in the ring of integers modulo n. Each commitment is computed

- either by performing operations of the type

$$R \equiv r^v \bmod n$$

where r is a random value produced by the random factor production means, r being such that $0 < r < n$.

- or by performing operations of the type

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0 < r_i < p_i$, each r_i belonging to a collection of random values $\{r_1, r_2, \ldots r_f\}$, produced by the random factor production means and then by applying the Chinese remainder method.

The witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d; each challenge d comprising m integers d_i hereinafter called elementary challenges.

- computation means, hereinafter called means for the computation of the responses D of the witness device, for the computation on the basis of each challenge d, of a response D:

• either by carrying out operations of the type:

$$D \equiv r \cdot Q_1^{d1} \cdot Q_2^{d2} \cdot \ldots Q_m^{dm} \mod n$$

• or by carrying out operations of the type:

$$D_i \equiv r_i \cdot Q_{i,1}^{d1} \cdot Q_{i,2}^{d2} \cdot \ldots Q_{i,m}^{dm} \mod p_i$$

and then by applying the Chinese remainder method.

The witness device also comprises transmission means to transmit one or more commitments R and one or more responses D. There are as many responses D as there are challenges d as there are commitments R. Each group of numbers R, d, D forming a triplet referenced {R, d, D}.

Case of the proof of the authenticity of an entity

In a first variant of embodiment, the system according to the invention is designed to prove the authenticity of an entity called a demonstrator to an entity called a controller.

Said system is such that it comprises a demonstrator device associated with a demonstrator entity. Said demonstrator device is interconnected with the witness device by interconnection means. It may especially take the form of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card.

Said system also comprises a controller device associated with the controller entity. Said controller device especially takes the form of a terminal or remote server. Said controller device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a computer communications network, to the demonstrator device.

Said system is used to execute the following steps:

• **Step 1: act of commitment R**

At each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified above. The witness device comprises means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means. The demonstrator device also comprises transmission means, hereinafter called the transmission means of the demonstrator, to transmit all or part of each commitment R to the controller device through the connection means.

• **Step 2: act of challenge d**

The controller device comprises challenge production means for the production, after receiving all or part of each commitment R, of the challenges d equal in number to the number of commitments R. The controller device also comprises transmission means, hereinafter called the transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means.

• **Step 3: act of response D**

The means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means. The means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified above.

• **Step 4: act of checking**

The transmission means of the demonstrator transmit each response D to the controller. The controller device also comprises:

- computation means, hereinafter called the computation means of the controller device,

- comparison means, hereinafter called the comparison means of the controller device.

First case : the demonstrator has transmitted a part of each commitment R.

If the transmission means of the demonstrator have transmitted a part of each commitment R, the computation means of the controller device, having m public values $G_1, G_2, \ldots Gm$, compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type:

$$R' \equiv D^v/G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} \bmod n.$$

The comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received.

Second case : the demonstrator has transmitted the totality of each commitment R

If the transmission means of the demonstrator have transmitted the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values $G_1, G_2, \ldots, G_m$, ascertain that each commitment R satisfies a relationship of the type:

$$R \equiv G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type:

$$R \equiv D^v/G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} \bmod n.$$

Case of the proof of the integrity of a message

In a second variant embodiment capable of being combined with the first one, the system according to the invention is designed to give proof to an entity, known as a controller of the integrity of a message M associated with an entity known as a demonstrator. Said system is such that it comprises a demonstrator device associated with the demonstrator entity. Said demonstrator device is

interconnected with the witness device by interconnection means. It may especially take the form of logic microcircuits in a nomad object, for example in the form of a microprocessor in a microprocessor-based bank card. Said system also comprises a controller device associated with the controller entity. Said controller device especially takes the form of a terminal or remote server. Said controller device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the demonstrator device.

Said system is used to execute the following steps:

• **Step 1: act of commitment R**

At each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified above. The witness device has means of transmission, hereinafter called transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means.

• **Step 2: act of challenge d**

The demonstrator device comprises computation means, hereinafter called the computation means of the demonstrator, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute at least one token T. The demonstrator device also comprises transmission means, hereinafter known as the transmission means of the demonstrator device, to transmit each token T through the connection means to the controller device. The controller device also has challenge production means for the production, after having received the token T, of the challenges d in a number equal to the number of commitments R. The controller device also has transmission means, hereinafter called the transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means.

• Step 3: act of response D

The means of reception of the challenges d of the witness device receive each challenge d coming from the demonstrator device through the interconnection means. The means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified above.

• Step 4: act of checking

The transmission means of the demonstrator transmit each response D to the controller. The controller device also comprises computation means, hereinafter called the computation means of the controller device, having m public values $G_1, G_2, \ldots, G_m$, to firstly compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} \cdot G_2^{d2} \cdot \ldots G_m^{dm} \cdot D^v \bmod n$$

or a relationship of the type:

$$R' \equiv D^v / G_1^{d1} \cdot G_2^{d2} \cdot \ldots G_m^{dm} \bmod n$$

and then, secondly, to compute a token T' by applying the hashing function h having as arguments the message M and all or part of each reconstructed commitment R'.

The controller device also has comparison means, hereinafter known as the comparison means of the controller device, to compare the computed token T' with the received token T.

Digital signature of a message and proof of its authenticity

In a third variant of an embodiment according to the invention, capable of being combined with one and/or the other of the first two embodiments, the system according to the invention is designed to prove the digital signature of a message M, hereinafter known as a signed message, by an entity called a signing entity.

The signed message comprises:

- the message M,
- the challenges d and/or the commitments R,
- the responses D.

Signing operation

Said system is such that it comprises a signing device associated with the signing entity. Said signing device is interconnected with the witness device by interconnection means and may especially take the form of logic microcircuits in a nomad object, for example in the form of a microprocessor in a microprocessor-based bank card.

Said system is used to execute the following steps:

• **Step 1: act of commitment R**

At each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified above. The witness device comprises means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the signing device through the interconnection means.

• **Step 2: act of challenge d**

The signing device comprises computation means, hereinafter called the computation means of the signing device, applying a hashing function h whose arguments are the message M and all or part of each commitment R to compute a binary train and extract, from this binary train, challenges d whose number is equal to the number of commitments R.

• **Step 3: act of response D**

The means for the reception of the challenges d of the witness device receive each challenge d coming from the signing device through the interconnection means. The means for computing the responses D of the witness device compute the responses D from the challenges d by applying the process specified above. The witness device comprises transmission means, hereinafter called means of transmission of the witness

device, to transmit the responses D to the signing device, through the interconnection means.

Checking operation :

To prove the authenticity of the message M, an entity known as the controller checks the signed message.

Said system comprises a controller device associated with the controller entity. Said controller device especially takes the form of a terminal or remote server. Said controller device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a computer communications network, to the signing device.

Said signing device associated with the signing entity comprises transmission means, hereinafter known as the transmission means of the signing device, for the transmission, to the controller device, of the signed message through the connection means. Thus the controller device has a signed message comprising:

- the message M,
- the challenges d and/or the commitments R,
- the responses D.

The controller device comprises:

- computation means hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device.

- **Case where the controller device has commitments R, challenges d, responses D**

If the controller device has commitments R, challenges d, responses D, the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1} \,.\, G_2^{d2} \,.\, \dots\, G_m^{dm} \,.\, D^v \bmod n$$

or relationships of the type:

$$R \equiv D^v/G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \mod n.$$

Then, the computation and comparison means of the controller device ascertain that the message M, the challenges d and the commitments R satisfy the hashing function:

$$d = h(\text{message}, R)$$

• Case where the controller device has challenges d and responses D

If the controller has challenges d and responses D, the computation means of the controller reconstruct, on the basis of each challenge d and each response D, commitments R' satisfying relationships of the type

$$R' \equiv G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \,.\, D^v \mod n$$

or relationships of the type:

$$R' \equiv D^v/G_1^{d1} \,.\, G_2^{d2} \,.\, \ldots\, G_m^{dm} \mod n.$$

Then the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function:

$$d = h\ (\text{message}, R')$$

• Case where the controller has commitments R and responses D

If the controller device has commitments R and responses D, the computation means of the controller device apply the hashing function and compute d' such that:

$$d' = h\ (\text{message}, R)$$

Then the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type:

$$R \equiv G_1^{d'1} \,.\, G_2^{d'2} \,.\, \ldots\, G_m^{d'm} \,.\, D^v \mod n$$

or relationships of the type:

$$R \equiv D^v/G_1^{d'1} \,.\, G_2^{d'2} \,.\, \ldots\, G_m^{d'm} \mod n.$$

Terminal Device

The invention also relates to a terminal device associated with an entity. The terminal device especially takes the form of a nomad object, for example the form of a microprocessor in a

microprocessor-based bank card, The terminal device is designed to prove the following to a controller device:

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity.

This proof is established by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of private values $Q_1, Q_2, \ldots Q_m$ and public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public module n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2).

Said module and said private and public values are related by relationships of the type

$$G_i \bullet Q_i^v \equiv 1 \mod n \text{ or } G_i \equiv Q_i^v \mod n$$

v designating a public exponent of the type

$$v = 2^k$$

where k is a security parameter greater than 1.

Said m public values G_i are the squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$ inferior to the f prime factors $p_1, p_2, \ldots p_f$; said f prime factors $p_1, p_2, \ldots p_f$ and/or said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied.

First condition

According to the first condition, each of the equations:

$$x^v \equiv g_i^2 \mod n \quad (1)$$

can be solved in x in the ring of integers modulo n.

Second condition

According to the second condition, in the case where $G_i \equiv Q_i^v \mod n$, among the m numbers q_i obtained by raising Q_i to the square modulo n, k-1 times, one of them is different from $\pm g_i$ (that is to say non trivial).

According to the second condition, in the case where $G_i \cdot Q_i^v \equiv 1 \mod n$, among the m numbers q_i obtained by raising the inverse of Q_i modulo n to the square modulo n, k-1 times, one of them is different from $\pm g_i$ (that is to say non trivial).

It is pointed out here that according to a current notation $\pm g_i$ represents the numbers g_i and $n-g_i$.

Third condition:

According to the third condition, among the 2 m equations:

$$x^2 \equiv g_i \bmod n \quad (2)$$

$$x^2 \equiv g_i \bmod n \quad (3)$$

at least one of them can be solved in x in the ring of integers modulo n.

Said terminal device comprises a witness device comprising a memory zone containing the f prime factors p_i and/or the parameters of the Chinese remainders of the prime factors and/or the public modulus n and/or the m private values Q_i and/or f.m components $Q_{i,j}$ ($Q_{i,j} = Q_j \bmod p_j$) of the private values Q_i and of the public exponent v.

The witness device also comprises:

- random value production means hereinafter called random value production means of the witness device,
- computation means, hereinafter called means for the computation of commitments R of the witness device in the ring of the integers modulo n.

Each commitment is computed

• either by performing operations of the type:

$$R \equiv r^v \bmod n$$

where r is a random value produced by the random value production means, r being such that $0<r<n$.

• or by performing operations of the type:

$$R_i \equiv r_i^v \bmod p_i$$

where r_i is a random value associated with the prime number p_i such that $0<r_i<p_i$, each r_i belonging to a collection of random values $\{r_1, r_2, \dots r_f\}$ produced by the random value production means, then by applying the Chinese remainder method.

The witness device also comprises:

- reception means hereinafter called the means for the reception of the challenges d of the witness device, to receive one or more challenges d, each challenge d comprising m integers d_i hereinafter called elementary challenges;

- computation means, hereinafter called means for the computation of the responses D of the witness device, for the computation, on the basis of each challenge d, of a response D,

- either by performing operations of the type:

$$D \equiv r \cdot Q_1^{d1} \cdot Q_2^{d2} \cdot \ldots Q_m^{dm} \mod n$$

- or by performing operations of the type:

$$D \equiv r_i \cdot Q_{i,1}^{d1} \cdot Q_{i,2}^{d2} \cdot \ldots Q_{i,m}^{dm} \mod p_i$$

and then by applying the Chinese remainder method.

Said witness device also comprises transmission means to transmit one or more commitments R and one or more responses D. There are as many responses D as there are challenges d as there are commitments R. Each group of numbers R, d, D forms a triplet referenced {R, d, D}.

Case of the proof of the authenticity of an entity

In a first embodiment variant, the terminal device according to the invention is designed to prove the authenticity of an entity called a demonstrator to an entity called a controller.

Said terminal device is such that it comprises a demonstrator device associated with a demonstrator entity. Said demonstrator device is interconnected with the witness device by interconnection means. It may especially take the form of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card.

Said demonstrator device also comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the controller device associated

with the controller entity. Said controller device especially takes the form of a terminal or remote server.

Said terminal device is used to execute the following steps:

• Step 1: act of commitment R

At each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified above.

The witness device has means of transmission, hereinafter called transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means. The demonstrator device also has transmission means, hereinafter called the transmission means of the demonstrator, to transmit all or part of each commitment R to the controller device, through the connection means

• Steps 2 and 3: act of challenge d, act of response D

The means of reception of the challenges d of the witness device receive each challenge d coming from the controller device through the connection means between the controller device and the demonstrator device and through the interconnection means between the demonstrator device and the witness device. The means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified above.

• Step 4: act of checking

The transmission means of the demonstrator transmit each response D to the controller device that carries out the check.

Case of the proof of the integrity of a message

In a second embodiment variant capable of being combined with the first embodiment, the terminal device according to the invention is designed to give proof to an entity, known as a controller, of the integrity of a message M

associated with an entity known as a demonstrator. Said terminal device is such that it comprises a demonstrator device associated with the demonstrator entity, said demonstrator device being interconnected with the witness device by interconnection means. It may especially take the form of logic microcircuits in a nomad object, for example the form of a microprocessor in a microprocessor-based bank card. Said demonstrator device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the controller device associated with the controller entity. Said controller device especially takes the form of a terminal or remote server.

Said terminal device is used to execute the following steps:

• Step 1: act of commitment R

At each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified above. The witness device has means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the demonstrator device through the interconnection means.

• Steps 2 and 3: act of challenge d, act of response D

The demonstrator device comprises computation means, hereinafter called the computation means of the demonstrator, applying a hashing function h whose arguments are the message M and all or part of each commitment R, to compute at least one token T. The demonstrator device also has transmission means, hereinafter known as the transmission means of the demonstrator device, to transmit each token T, through the connection means, to the controller device.

Said controller device, after having received the token T, produces challenges d in a number equal to the number of commitments R

The means of reception of the challenges d of the witness device receive each challenge d coming from the controller device through the interconnection means between the controller device and the demonstrator device and through the interconnection means between the demonstrator device and the witness device. The means of computation of the responses D of the witness device compute the responses D from the challenges d by applying the process specified above.

- **Step 4: act of checking**

The transmission means of the demonstrator send each response D to the controller device which performs the check.

Digital signature of a message and proof of its authenticity

In a third embodiment variant, capable of being combined with either one of the first two, the terminal device according to the invention is designed to produce the digital signature of a message M, hereinafter known as the signed message, by an entity called a signing entity.

The signed message comprises:

- the message M,
- the challenges d and/or the commitments R,
- the responses D.

Said terminal device is such that it comprises a signing device associated with the signing entity. Said signing device is interconnected with the witness device by interconnection means. It may especially take the form of logic microcircuits in a nomad object, fox example the form of a microprocessor in a microprocessor-based bank card. Said signing device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to the controller device associated with the controller entity. Said

controller device especially takes the form of a terminal or remote server.

Signing operation :

Said terminal device is used to execute the following steps:

- **Step 1: act of commitment R**

At each call, the means of computation of the commitments R of the witness device compute each commitment R by applying the process specified above. The witness device comprises means of transmission, hereinafter called the transmission means of the witness device, to transmit all or part of each commitment R to the signing device through the interconnection means.

- **Step 2: act of challenge d**

The signing device comprises computation means, hereinafter called the computation means of the signing device, applying a hashing function h whose arguments are the message M and all or part of each commitment R, to compute a binary train and extract, from this binary train, challenges d whose number is equal to the number of commitments R.

- **Step 3: act of response D**

The means for the reception of the challenges d receive each challenge d coming from the signing device through the interconnection means. The means for computing the responses D of the witness device compute the responses D from the challenges d by applying the process specified above. The witness device comprises transmission means, hereinafter called means of transmission of the witness device, to transmit the responses D to the signing device, through the interconnection means.

Controller device

The invention also relates to a controller device. The controller device may especially take the form of a terminal or remote server associated with a controller entity. The controller device is designed to check :

- the authenticity of an entity and/or
- the integrity of a message M associated with this entity.

This proof is established by means of all or part of the following parameters or derivatives of these parameters:

- m pairs of public values $G_1, G_2, \ldots G_m$ (m being greater than or equal to 1),
- a public modulus n constituted by the product of f prime factors $p_1, p_2, \ldots p_f$ (f being greater than or equal to 2), unknown to the controller device and the associated controller entity.

Said modulus and said private and public values are related by relationships of the type

$$G_i \,.\, Q_i^v \equiv 1 \,.\, \text{mod } n \text{ or } G_i \equiv Q_i^v \text{ mod } n$$

v designating a public exponent of the type:

$$v = 2^k$$

where k is a security parameter greater than 1.

Said m public values G_i being the squares g_i^2 of m distinct base numbers $g_1, g_2, \ldots g_m$ inferior to the f prime factors $p_1, p_2, \ldots p_f$; said f prime factors $p_1, p_2, \ldots p_f$ and/or said m base numbers $g_1, g_2, \ldots g_m$ being produced such that the following conditions are satisfied.

First condition

According to the first condition, each of the equations:

$$x^v \equiv g_i^2 \text{ mod } n \quad (1)$$

can be solved in x in the ring of integers modulo n.

Second condition

According to the second condition, in the case where $G_i \equiv Q_i^v$. mod n, among the m numbers q_i obtained by raising Q_i to the square modulo n, k-1 rank times, one of them is different from $\pm g_i$ (that is to say non trivial).

According to the second condition, in the case where $G_i \,.\, Q_i^v \equiv 1$ mod n, among the m numbers q_i obtained by raising the inverse of Q_i modulo n to the square modulo n, k-1 rank times, one of them is different from $\pm g_i$ (that is to say non trivial).

It is pointed out here that according to a current notation $\pm g_i$ represents the numbers g_i and $n-g_i$.

Third condition

According to the third condition, among the 2m equations:

$$x^2 = g_i \bmod n \quad (2)$$
$$x^2 = g_i \bmod n \quad (3)$$

at least one of them can be solved in x in the ring of integers modulo n.

Case of the proof of the authenticity of an entity

In a first embodiment variant, the controller device according to the invention is designed to prove the authenticity of an entity called a demonstrator and an entity called a controller.

Said controller device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a demonstrator device associated with the demonstrator entity.

Said controller device is used to execute the following steps:

- **Steps 1 and 2: act of commitment R, act of challenge d**

Said controller device also has means for the reception of all or part of the commitments R coming from the demonstrator device through the connection means.

The controller device comprises challenge production means for the production, after receiving all or part of each commitment R, of the challenges d in a number equal to the number of commitments R, each challenge d comprising m integers d_i hereinafter called elementary challenges.

The controller device also comprises transmission means, hereinafter called transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means.

• Steps 3 and 4: act of response D, act of checking

The controller device also comprises:

- means for the reception of the responses D coming from the demonstrator device, through the connection means,
- computation means, hereinafter called the computation means of the controller device,
- comparison means, hereinafter called the comparison means of the controller device.

• First case: the demonstrator has transmitted a part of each commitment R.

If the reception means of the controller device have received a part of each commitment R, the computation means of the controller device, having m public values $G_1, G_2, \dots G_m$, compute a reconstructed commitment R', from each challenge d and each response D, this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} \cdot G_2^{d2} \dots G_m^{dm} \cdot D^v \bmod n$$

or a relationship of the type:

$$R' \equiv D^v / G_1^{d1} \cdot G_2^{d2} \cdot \dots G_m^{dm} \bmod n.$$

The comparison means of the controller device compare each reconstructed commitment R' with all or part of each commitment R received.

• Second case: the demonstrator has transmitted the totality of each commitment R

If the reception means of the controller device have received the totality of each commitment R, the computation means and the comparison means of the controller device, having m public values $G_1, G_2, \dots G_m$, ascertain that each commitment R satisfies a relationship of the type:

$$R' \equiv G_1^{d1} \cdot G_2^{d2} \dots G_m^{dm} \cdot D^v \bmod n$$

or a relationship of the type:

$$R' \equiv D^v / G_1^{d1} \cdot G_2^{d2} \cdot \dots G_m^{dm} \bmod n.$$

Case of the proof of the integrity of a message

In a second embodiment variant capable of being combined with the first, the controller device according to the invention is designed to prove the integrity of a message M associated with an entity known as a demonstrator.

Said controller device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a demonstrator device associated with the demonstrator entity.

Said controller device is used to execute the following steps:

- **Steps 1 and 2: act of commitment R, act of challenge d**

Said controller device has means for the reception of tokens T coming from the demonstrator device through the connection means. The controller device has challenge production means for the production, after having received the token T, of challenges d in a number equal to the number of commitments R, each challenge d comprising m integers d_i hereinafter called elementary challenges. The controller device also has transmission means, hereinafter called the transmission means of the controller, to transmit the challenges d to the demonstrator through the connection means.

- **Steps 3 and 4: act of response D, act of checking**

Said controller device also comprises means for the reception of the responses D coming from the demonstrator device, through the connection means. Said controller device also comprises computation means, hereinafter called the computation means of the controller device, having m public values G_1, G_2, ... G_m, to first of all compute a reconstructed commitment R', from each challenge d and each response D,

this reconstructed commitment R' satisfying a relationship of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} \ldots G_m^{dm} . D^v \bmod n$$

or a relationship of the type:

$$R' \equiv D^v / G_1^{d1} . G_2^{d2} . \ldots G_m^{dm} \bmod n$$

and then, secondly, to compute a token T' by applying a hashing function h having as arguments the message M and all or part of each reconstructed commitment R'.

The controller device also has comparison means, hereinafter called the comparison means of the controller device to compare the computed token T' with the received token T.

Digital signature of a message and proof of its authenticity

In a third embodiment variant, capable of being combined with either one or the other of the first two embodiments, the controller device according to the invention is designed to prove the authenticity of the message M by checking a signed message by means of an entity called a controller.

The signed message, sent by a signing device associated with a signing entity having a hashing function h (message, R) comprises:

- the message M,
- the challenges d and/or the commitments R,
- the responses D.

Checking operation

Said controller device comprises connection means for its electrical, electromagnetic, optical or acoustic connection, especially through a data-processing communications network, to a signing device associated with the signing entity. Said controller device receives the signed message from the signing device, through the connection means.

The controller device comprises:

- computation means, hereinafter called the computation means of the controller device,

- comparison means, hereinafter called the comparison means of the controller device.

• **Case where the controller device has commitments R, challenges d, responses D :**

If the controller device has commitments R, challenges d, responses D, the computation and comparison means of the controller device ascertain that the commitments R, the challenges d and the responses D satisfy relationships of the type

$$R \equiv G_1^{d1} . G_2^{d2} ....G_m^{dm} . D^v \bmod n$$

or relationships of the type:

$$R \equiv D^v/G_1^{d1} . G_2^{d2} . ... G_m^{dm} \bmod n.$$

Then the computation and comparison means of the controller device ascertain that the message M, the challenges d and the commitments R satisfy the hashing function:

$$d = h\ (\text{message}, R)$$

• **Case where the controller device has challenges d and responses D :**

If the controller device has challenges d and responses D, the computation means of the controller device can, on the basis of each challenge d and each response D, compute commitments R' satisfying relationships of the type:

$$R' \equiv G_1^{d1} . G_2^{d2} ....G_m^{dm} . D^v \bmod n$$

or relationships of the type:

$$R' \equiv D^v/G_1^{d1} . G_2^{d2} . ... G_m^{dm} \bmod n$$

and then the computation and comparison means of the controller device ascertain that the message M and the challenges d satisfy the hashing function

$$d = h(\text{message}, R')$$

• **Case where the controller device has commitments R and responses D :**

If the controller device has commitments R and responses D, the computation means of the controller device apply the hashing function and compute d' such that

$$d' = h\ (\text{message},\ R)$$

and then the computation and comparison means of the controller device ascertain that the commitments R, the challenges d' and the responses D satisfy relationships of the type

$$R \equiv G_1^{d'1} \,.\, G_2^{d'2} \,.\, \ldots G_m^{d'm} \,.\, D^v \bmod n$$

or relationships of the type:

$$R \equiv D^v / G_1^{d'1} \,.\, G_2^{d'2} \,.\, \ldots\, G_m^{d'm} \bmod n.$$

Description of the invention

The goals of the GQ methods are the dynamic authentication of entities and messages together with the digital signature of messages. These are methods "without transfer of knowledge". An entity proves that it knows one or several private numbers. Another entity checks; it knows the corresponding public number or numbers. The proving entity wants to convince the checking entity without revealing the private number or numbers, in such a way as to be able to use them as many times as needed.

Each GQ method depends on a public modulus composed of secret high prime numbers. A public exponent v and a public modulus n together form a verification key $<v,n>$ meaning "raise to the power v modulo n" and implement by means of one or several generic equations, all of the same type, direct: $G \equiv Q^v \pmod n$ or inverse $GxQ^v \equiv 1 \pmod n$. The type has an effect on the operation of the calculations within the checking entity, but not within the proving entity; in fact, the security analyses confound the two types. Each generic equation linked to a public number G and a private number Q together form a pair of numbers $\{G,Q\}$. To resume, each GQ method implements one or several pairs of numbers $\{G,Q\}$ for the same key $<v,n>$.

A classic version of the GQ method, here called GQ1, uses an RSA digital signature mechanism. The verification key $<v,n>$ is then an RSA public key where the odd exponent v is preferably a prime number. Each GQ1 method in general uses a single pair of numbers $\{G,Q\}$: the public number G is deduced from identification data according to a format mechanism which is an integral part of the RSA digital signature technology. The private number Q or its inverse modulo n is an RSA signature for identification data. The proving entity shows its knowledge of an RSA signature of its own identification data and this proof does not reveal the signature which therefore remains secret, to be used as many times as needed.

The GQ1 methods generally implement two levels of keys: the RSA private signing key is reserved for an authority accrediting entities distinguishing themselves one from the other by identification data. Such a mechanism is said to be "identity based". Thus, an emitter of chip cards uses his RSA private key for the emission of each card to calculate a private number Q which is inscribed as diversified private key in the card; or furthermore, a client on a network of computers uses his RSA private key when entering each session to calculate a private number Q which will be the ephemeral private key of the client during the session. The proving entities, chip cards or clients in session, know an RSA signature of their identification data; they do not know the RSA private key which, in the key hierarchy, is at the next higher level. Nonetheless, dynamic authentication of entities by GQ1 with a modulus of 768 bits at the level of an authority requires almost the same work load as a dynamic authentication of entities by RSA with a modulus of 512 bits with three prime factors at the level of each entity, which allows the proving entity to use the Chinese remainder technique by calculating a modulo result for each of the prime factors before calculating a modulo result for their product.